揺らぎ解析七転八起の道中記

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謝辞
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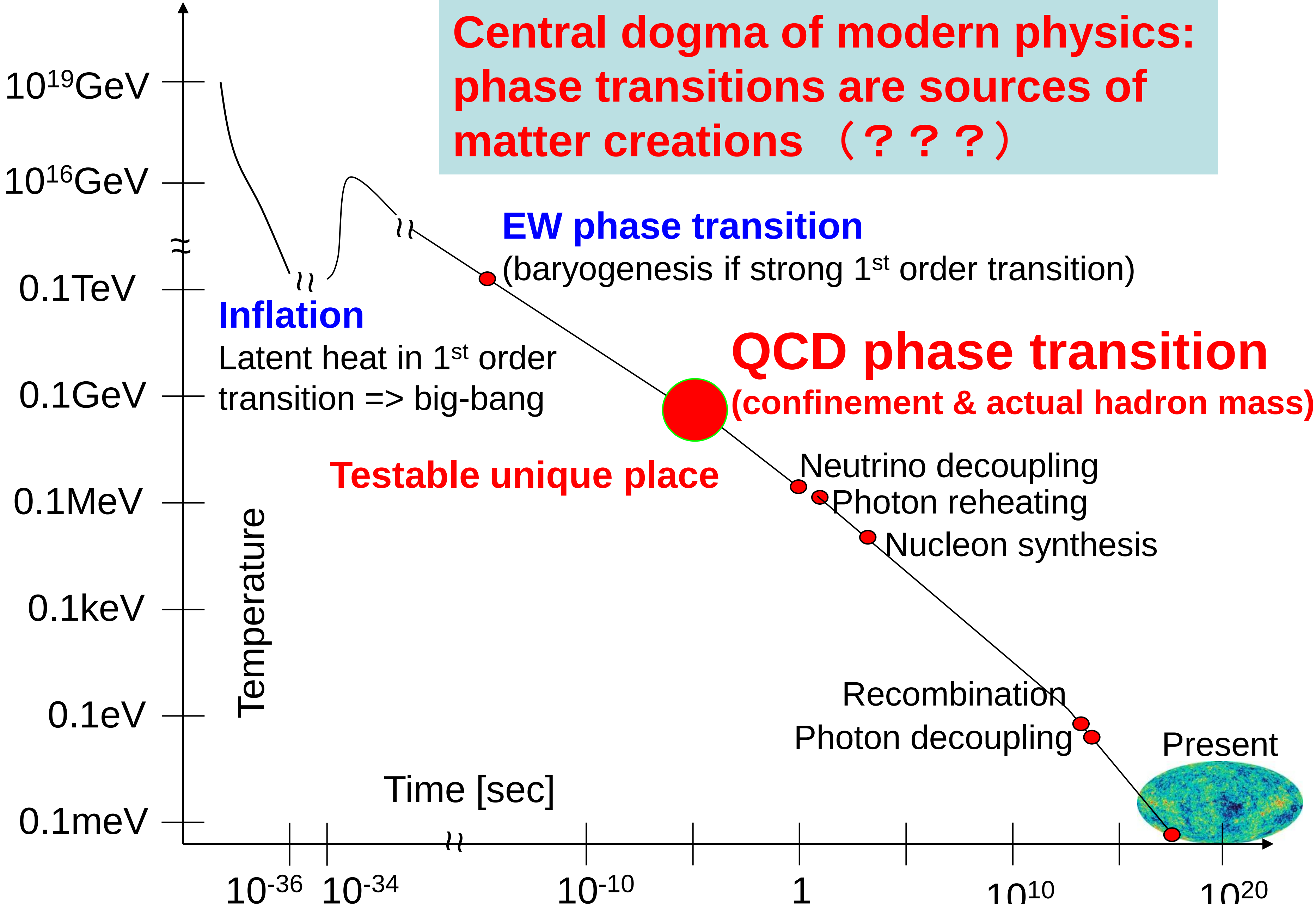
1. 真空の相転移なんて存在するのか？
2. 事象毎の揺らぎ解析（例として、DCCのようなものの探索）
3. 統計力学的な揺らぎ解析への転身
4. 事象毎の揺らぎ解析への回帰

Phase transitions in the early universe

Central dogma of modern physics: phase transitions are sources of matter creations （？？？）



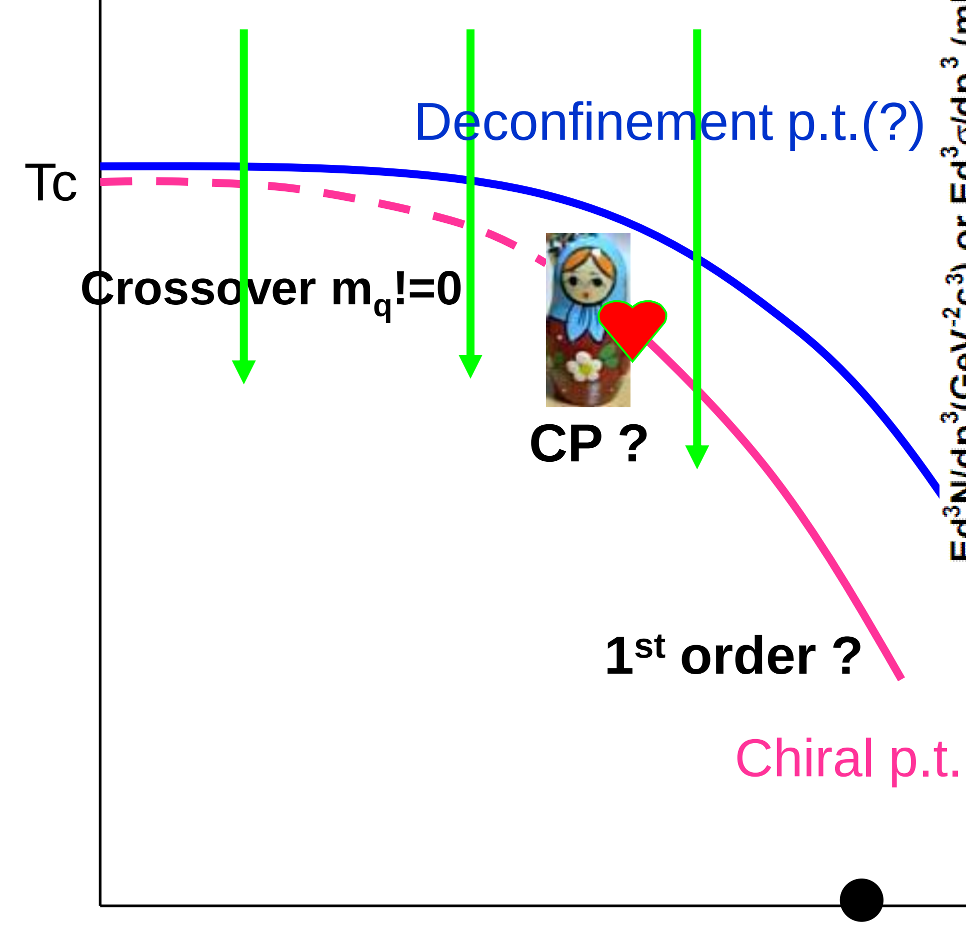
Conjectured QCD phase diagram

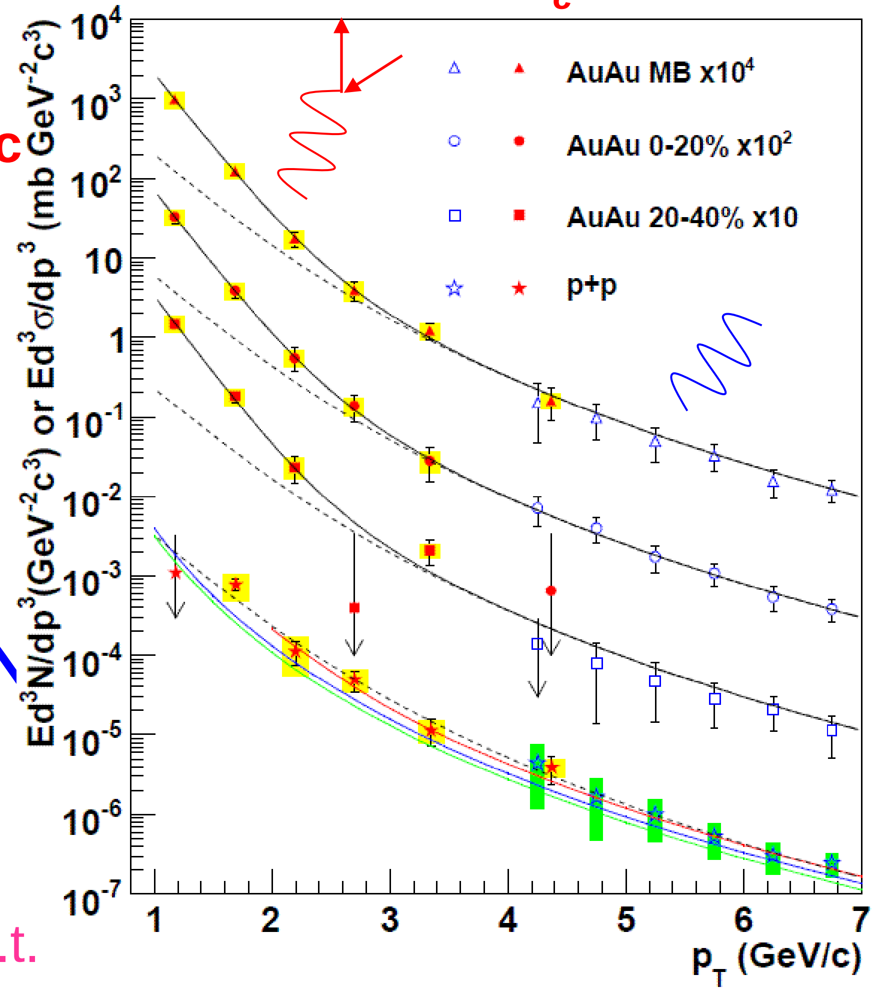
- High opacity state
- Bulk matter flow with quark d.o.f
- High temperature state

$T=221\pm23(stat)\pm18(sys)$

Lattice result T_c~170MeV

RHIC can approach from well above Tc





PHENIX thermal photon measurement

Imaging phase transition in EO crystal

Large electro-optical coefficient of 10^4 pm/V（Typically order of pm/V)

Fast rise and not too long duration time compared to effective impact time

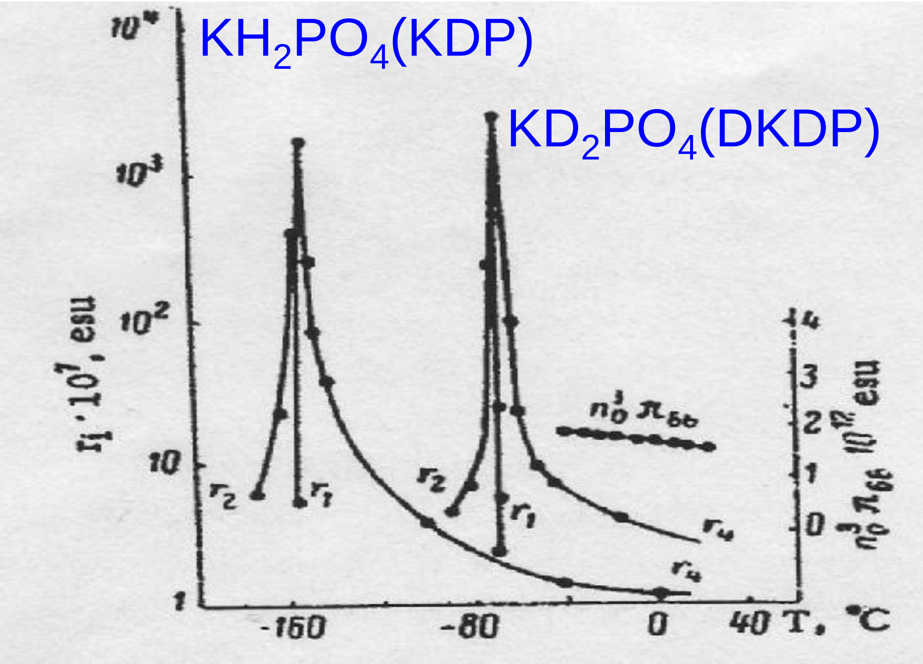


Fig. 1. Curves of electro-optical coefficients of PDP and DPDP crystals and piezo-optical constant $n_0^3 \pi_{66}$ for DPDP crystal versus temperature.

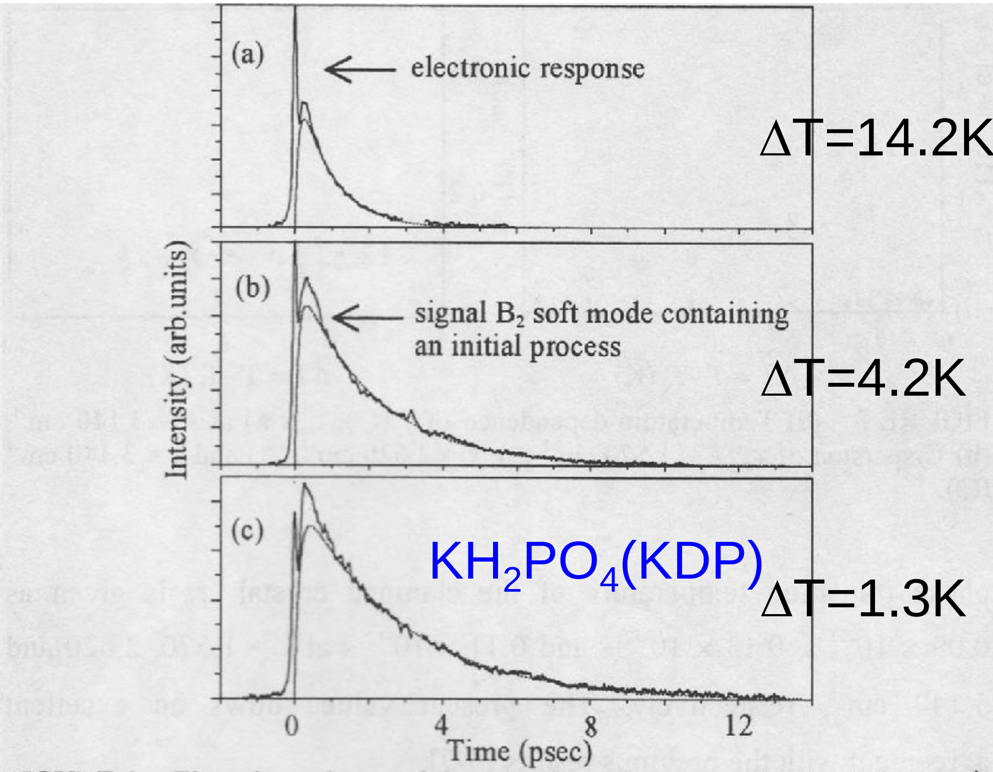
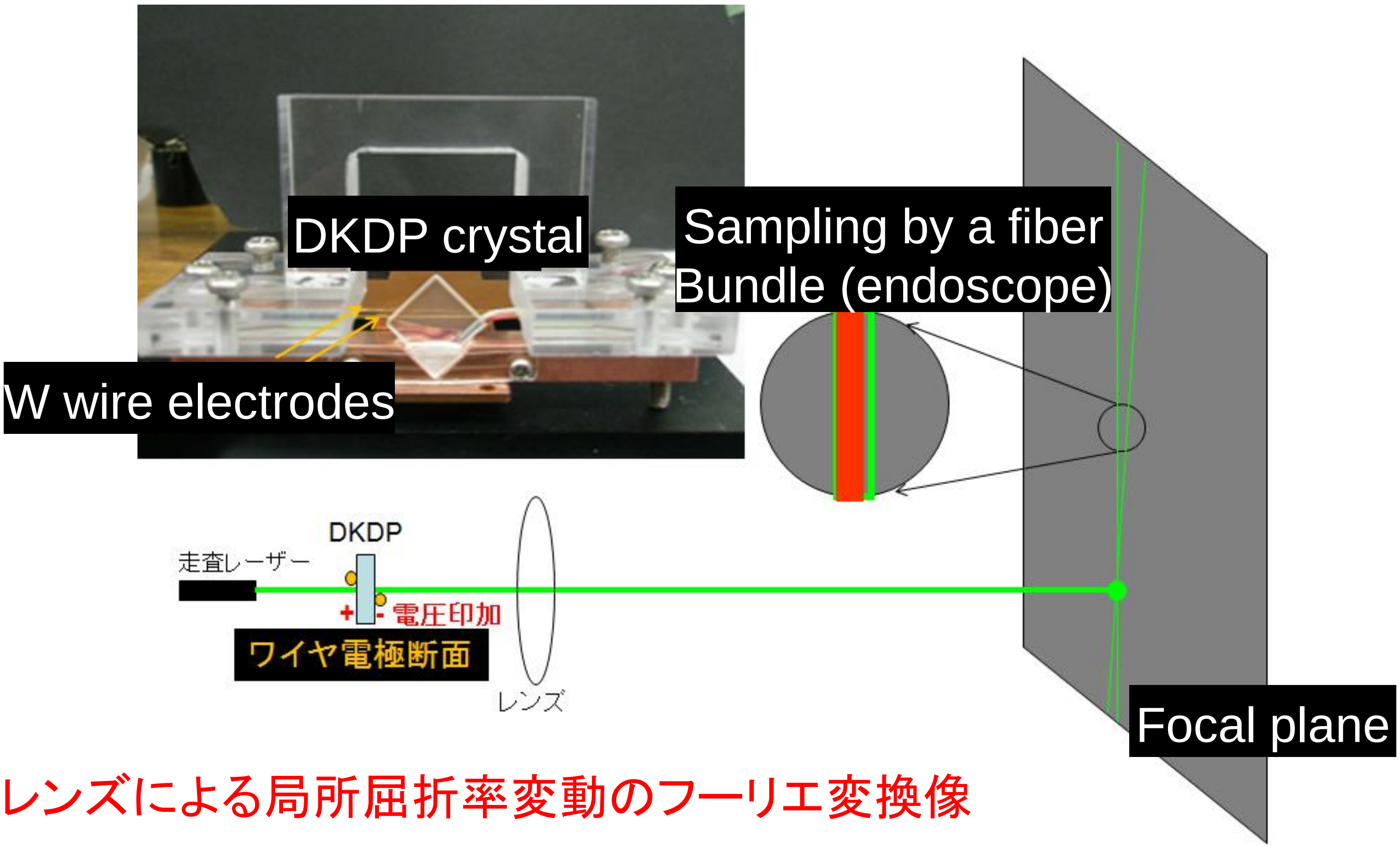


FIGURE 2 Time dependence of the B_2 soft mode observed at $k = 3{,}140$ cm^{-1}. Dashed lines are fit results by MDM. Temperature difference $\Delta T = T - T_c$ of each data is (a): $\Delta T = 14.2$ K, (b): $\Delta T = 4.2$ K, (c): $\Delta T = 1.3$ K.

Soviet Physics – Solid State Vol.8, No. 11 (1967) 2758-2760

Ferroelectrics, 2002 Vol.272, pp. 57-62

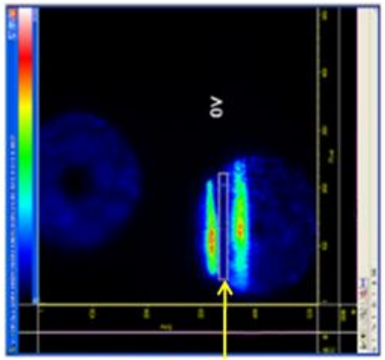
Fourier transform in spatial frequency

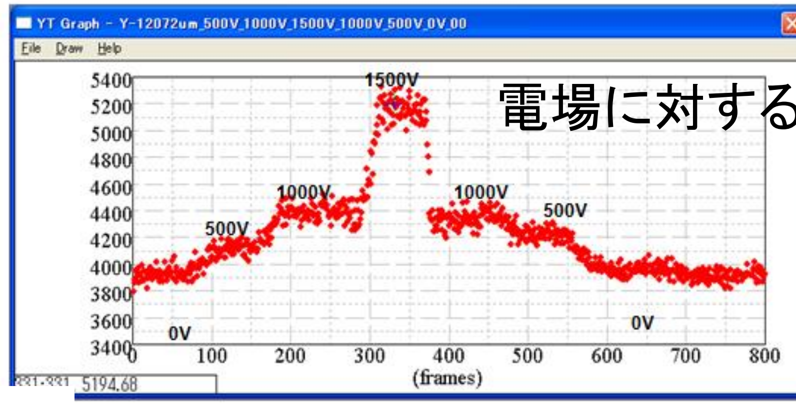


レンズによる局所屈折率変動のフーリエ変換像

Electro-optic response to static electric field

At room temperature



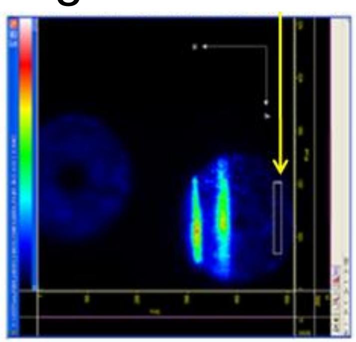


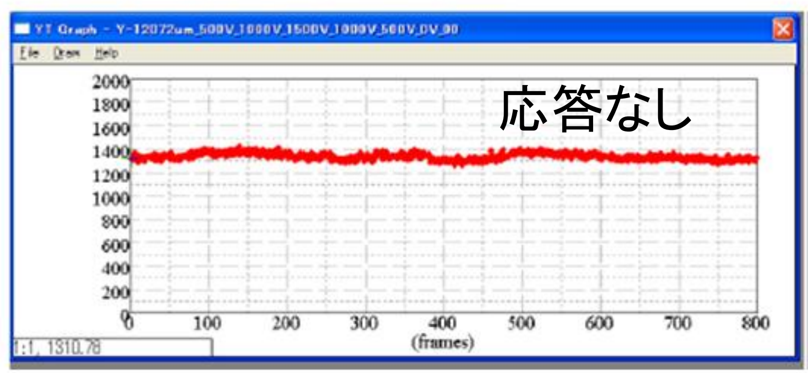
電場に対する局所応答

Sampling between two wires

Volt

Sampling far from wires





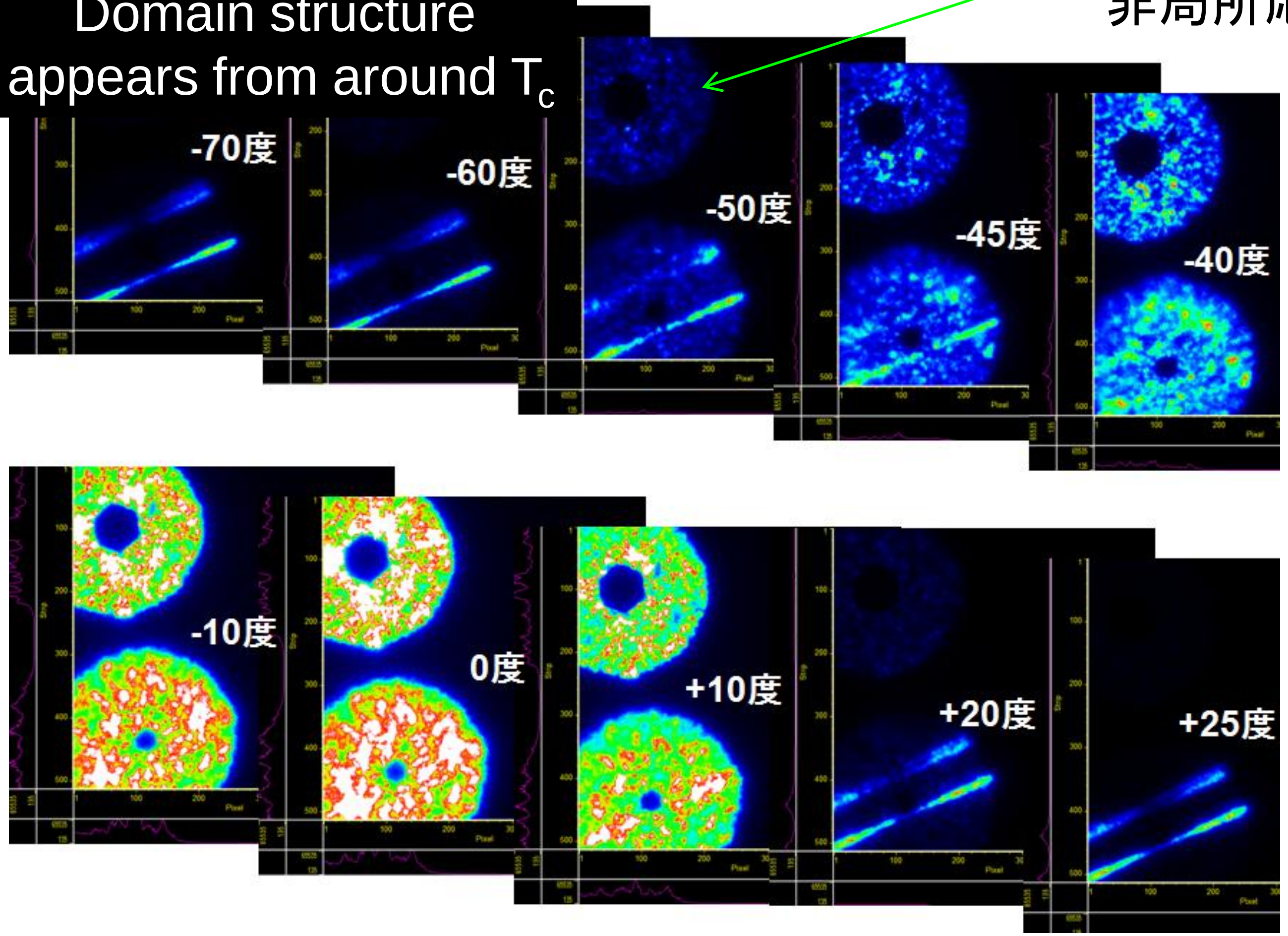
応答なし

Volt

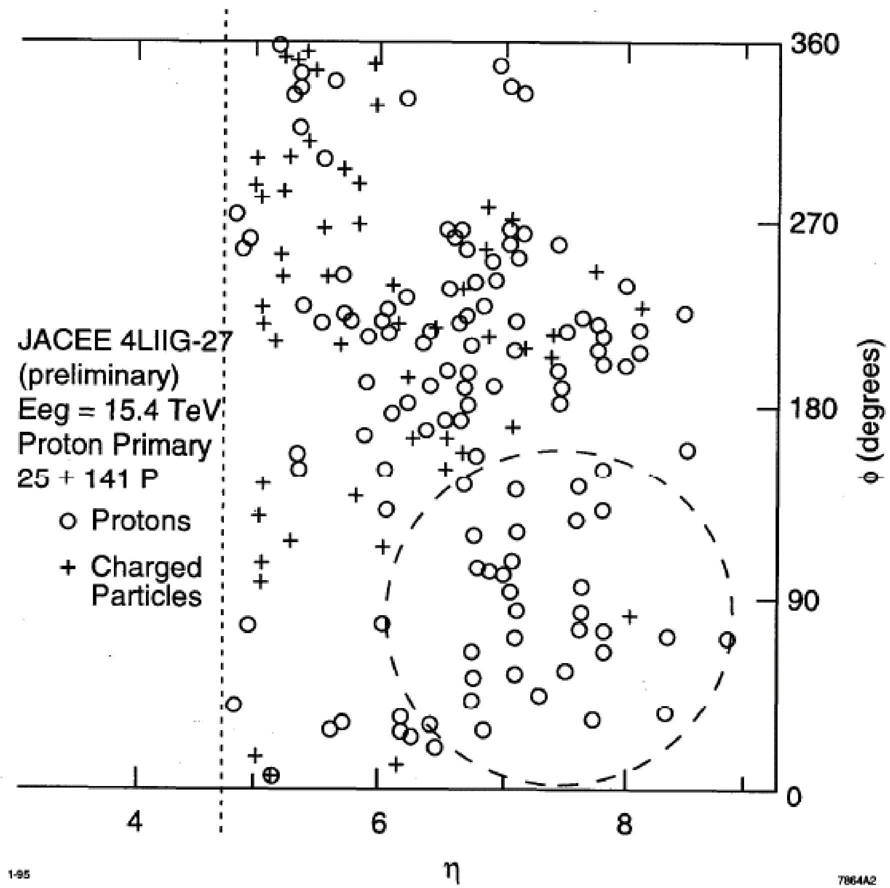
Demonstration of phase transitions in DKDP crystal

Domain structure appears from around T_c

相転移温度近傍での
非局所応答の出現



Centauro event in high-energy cosmic rays

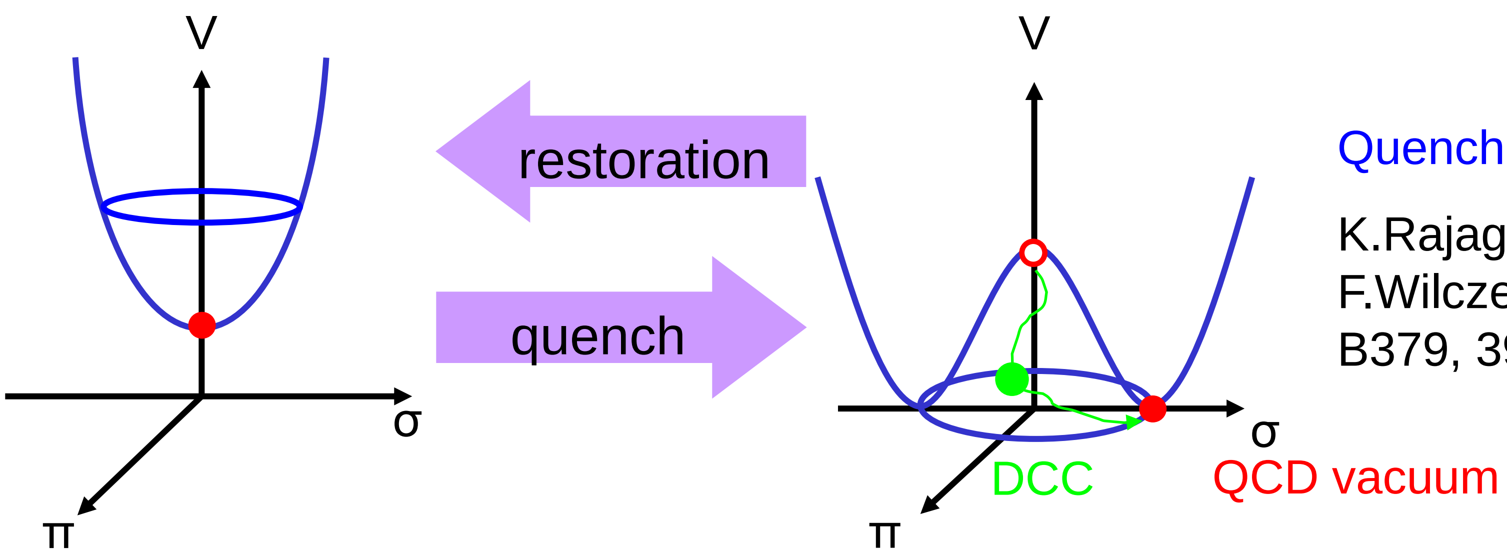


O: Photon

+ : Charged Particle

J. J. Lord and J. Iwai. Int. Conference on High Energy Physics, TX, 1992

Disoriented Chiral Condensate



Quench Mechanism

K.Rajagopal and F.Wilczek : Nucl. Phys. B379, 395 (1993)

Linear sigma model

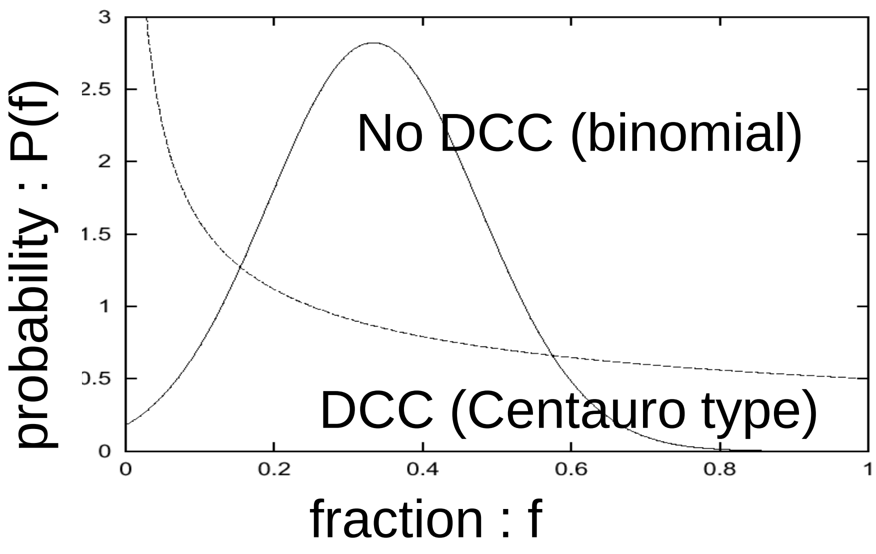
$$\phi_i = (\sigma, \vec{\pi})$$

$$L = \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i - \frac{1}{4}\left(\phi^2 - v^2\right)^2 + H\sigma$$

Initially proposed search Strategy

$$fraction: f = \frac{n_{\pi^0}}{n_{\pi^0} + n_{\pi^+} + n_{\pi^-}}$$

$$probability: P(f)df = \frac{1}{2\sqrt{f}} df$$



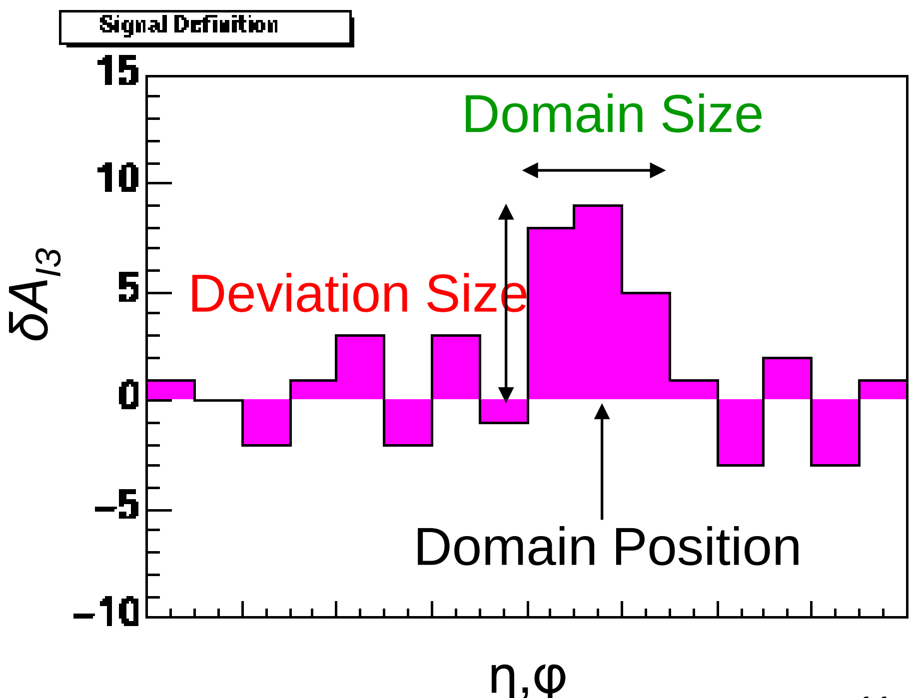
Focus on asymmetry with variable domain size

Define an asymmetry between number of charged tracks and neutral clusters in event-by-event base as a function of subdivided η-ϕ phase spaces normalized by one standard deviation for a given multiplicity class.

$$\delta A_{I_3}(\Delta\eta\Delta\phi) \equiv \frac{N_{\pi^\pm}(\Delta\eta\Delta\phi) - N_\gamma(\Delta\eta\Delta\phi)}{\sqrt{N_{\pi^\pm} + N_\gamma}}$$

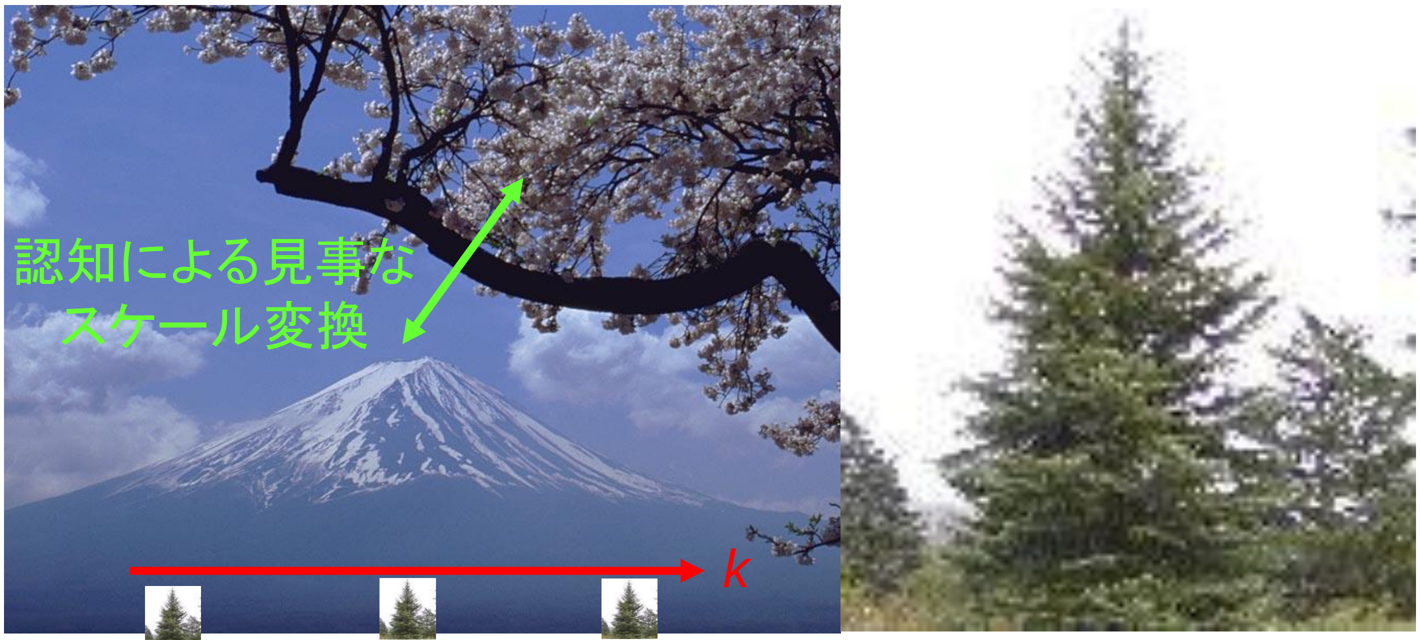
$$\approx \frac{N_{ch}(\Delta\eta\Delta\phi) - N_\gamma(\Delta\eta\Delta\phi)}{\sqrt{N_{ch} + N_\gamma}}$$

Domain size and domain position of largely deviated regions can be obtained at the same time by using Multi Resolution Analysis (MRA) technique.



ウェーブレット解析入門

有限幅の波束を基礎にするが故に、相対関係を刻むことが可能になる。



$$Mt.Fuji = \alpha\psi(\frac{k_0}{a_{Fuji}}) + \beta\psi(\frac{k_0 - 1}{a_{Fuji}}) + \gamma\psi(\frac{k_0 - 2}{a_{Fuji}})\cdots$$

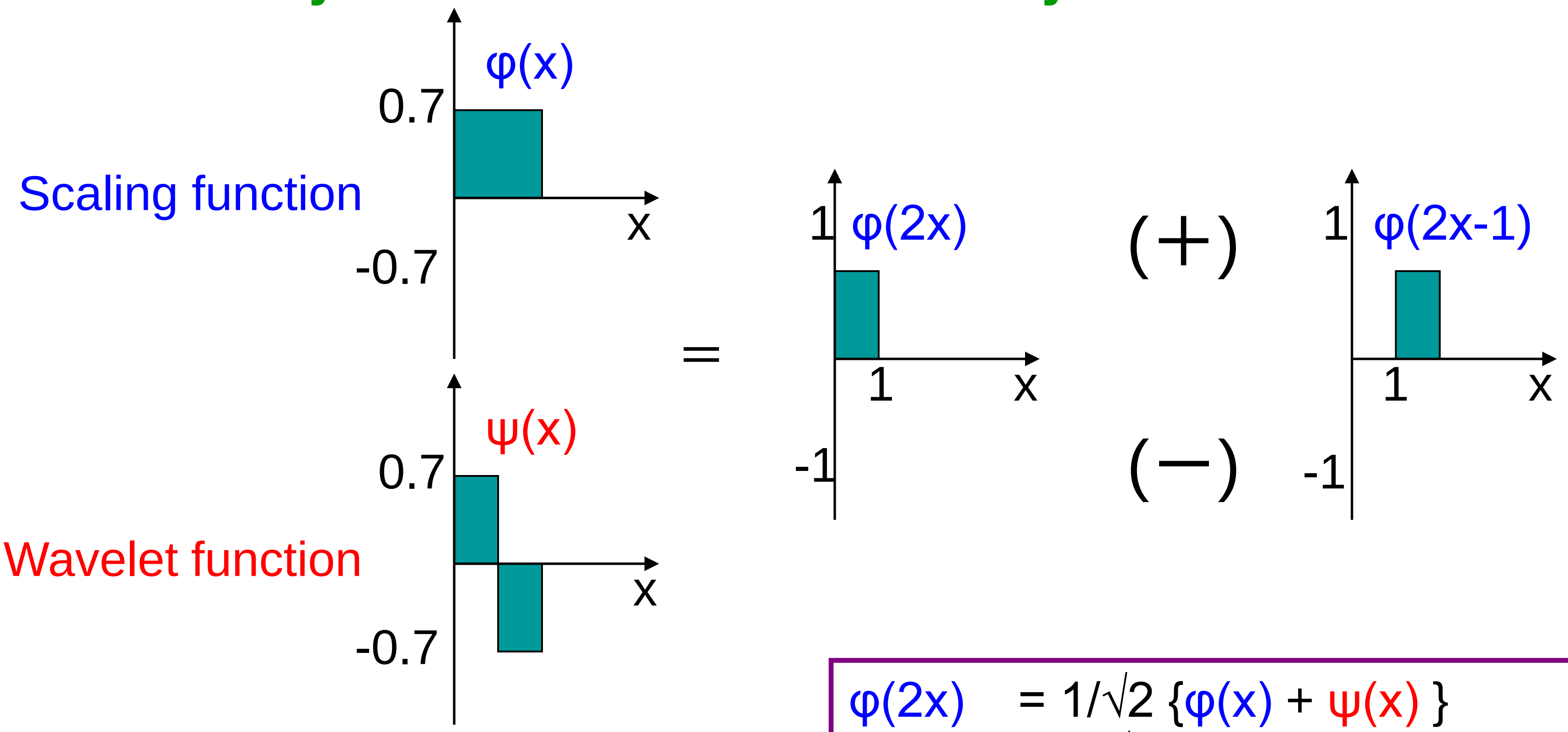
$$Tree = \psi(\frac{k_0}{a_{Tree}})$$

スケールaは有限で、$a_{Fuji} << a_{Tree}$

無限遠まで続く波の重ね合わせで記述するフーリエ積分では、富士山の形は表現できるが、それが木々の連なりで構成されていることは表現できない。

Multi Resolution Analysis (wavelet)

Level j-1 : 2^{j-1} bins   Level j : 2^{j} bins



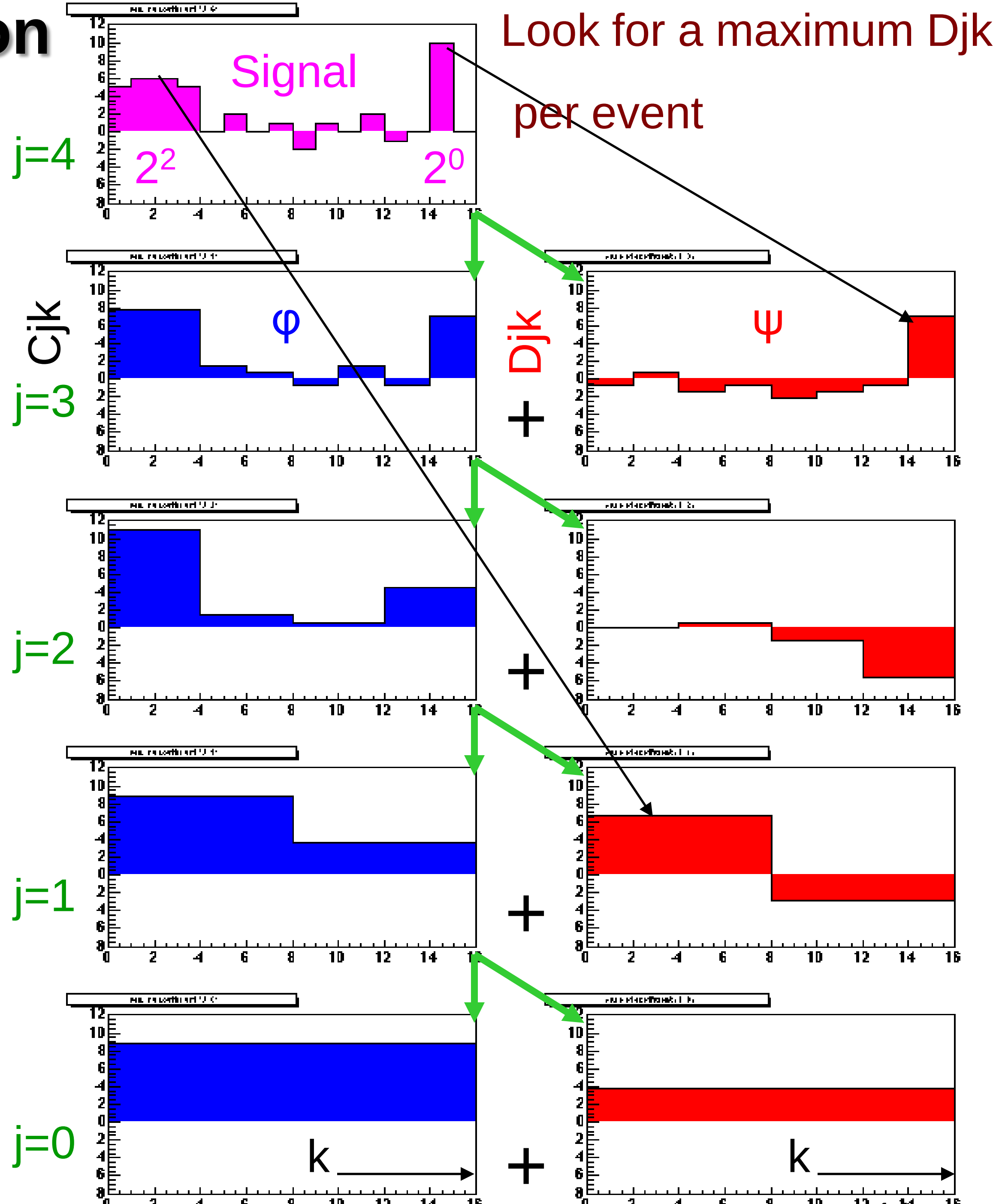
$\varphi(2x) = 1/\sqrt{2}\,\{\varphi(x) + \psi(x)\}$

$\varphi(2x-1) = 1/\sqrt{2}\,\{\varphi(x) - \psi(x)\}$

Total number of bins is 2^{j}

Level j represents a resolution level

Signal Decomposition



j : resolution level

k : k-th bin

Cjk : coefficients of φ

Djk : coefficients of ψ

$2^4/2^j$ → Domain Size

k → Domain Position

Cjk → Deviation Size

Djk → used to pick up k

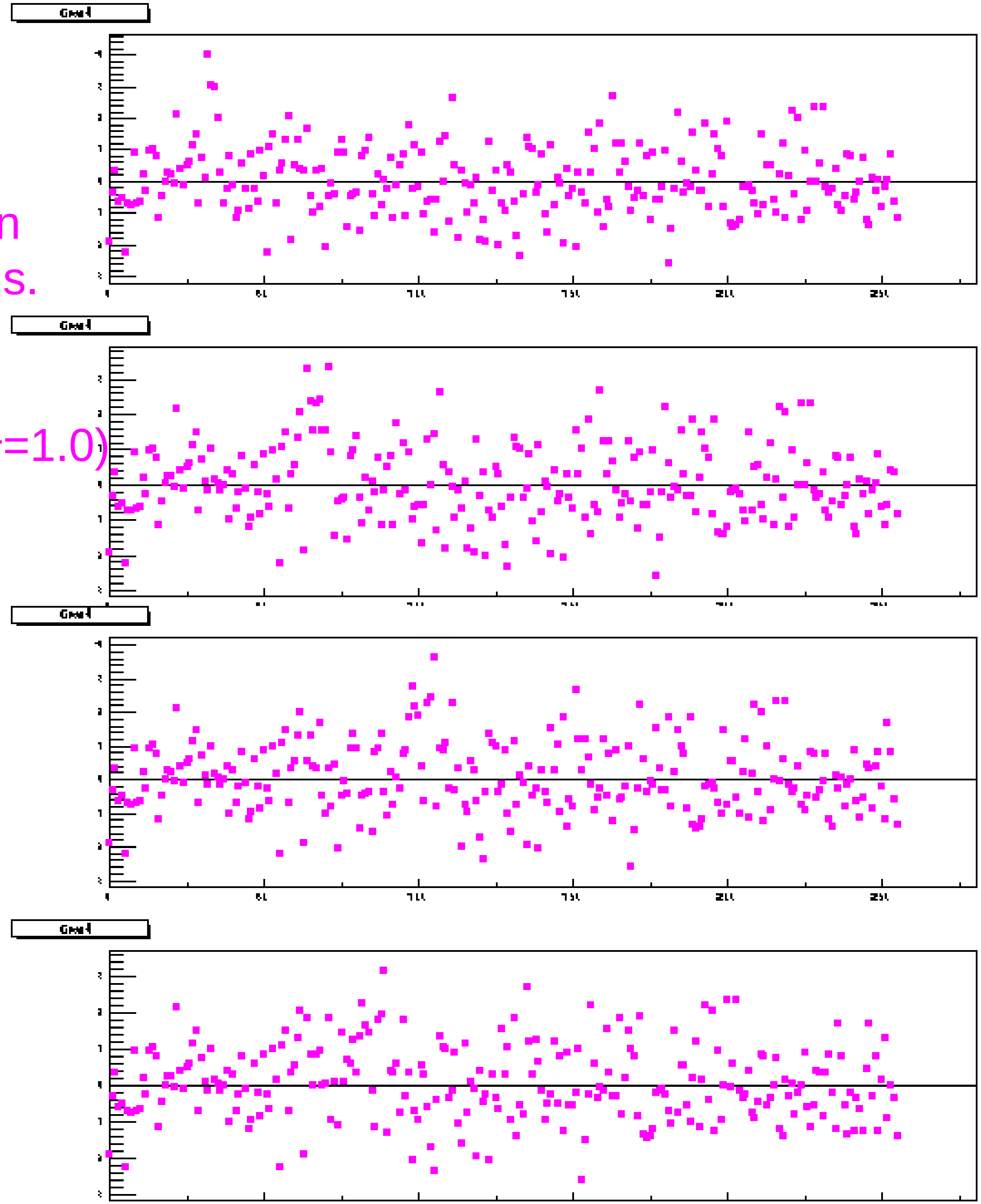
Quiz

Pink dots are distributed around 0 based on Gaussian (mean=0, σ=1.0) over 2^8 bins.

A single domain is hidden with Gaussian (mean=Nσ, σ=1.0)

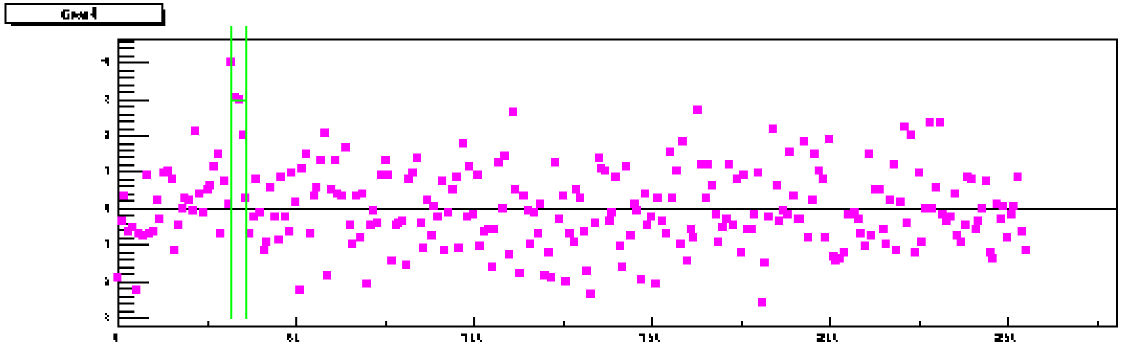
Where is an anomalous domain?

What is the domain size?

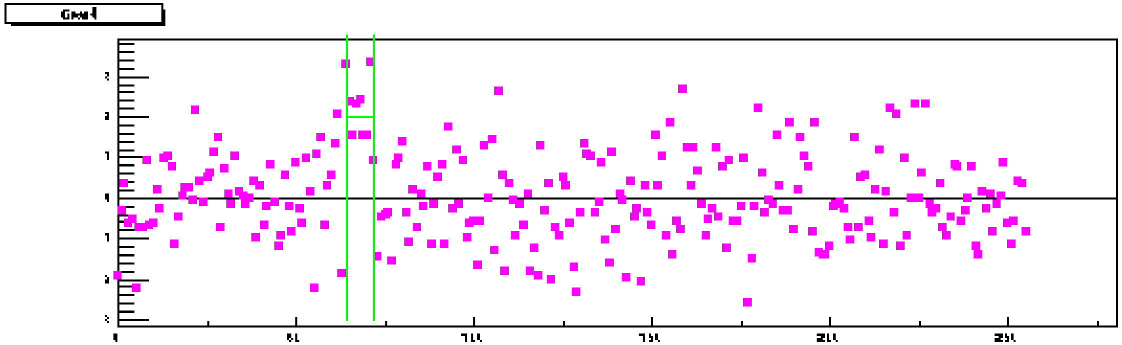


Answers

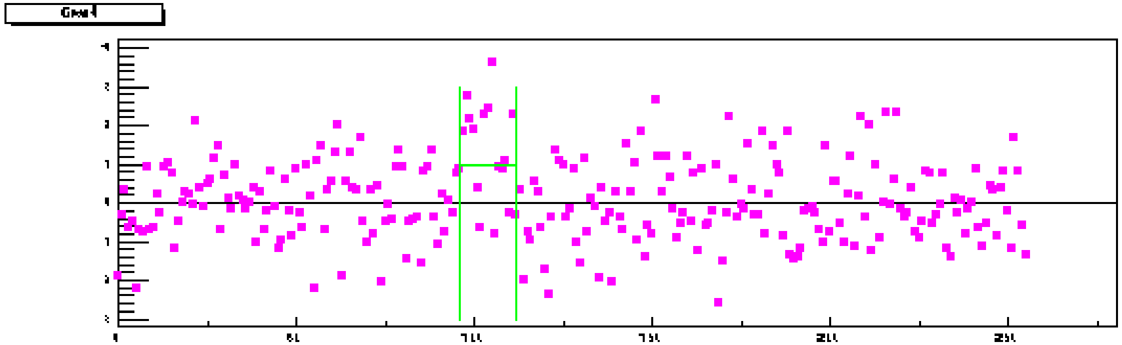
Nσ=3 j=5



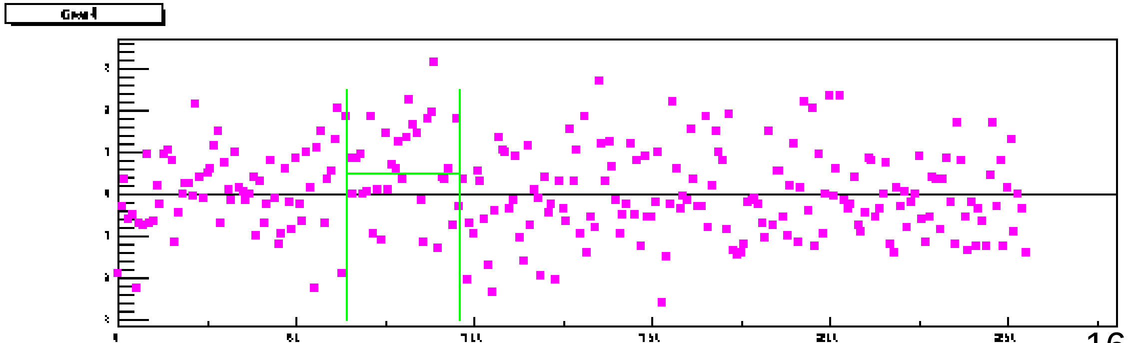
Nσ=2 j=4



Nσ=1 j=3

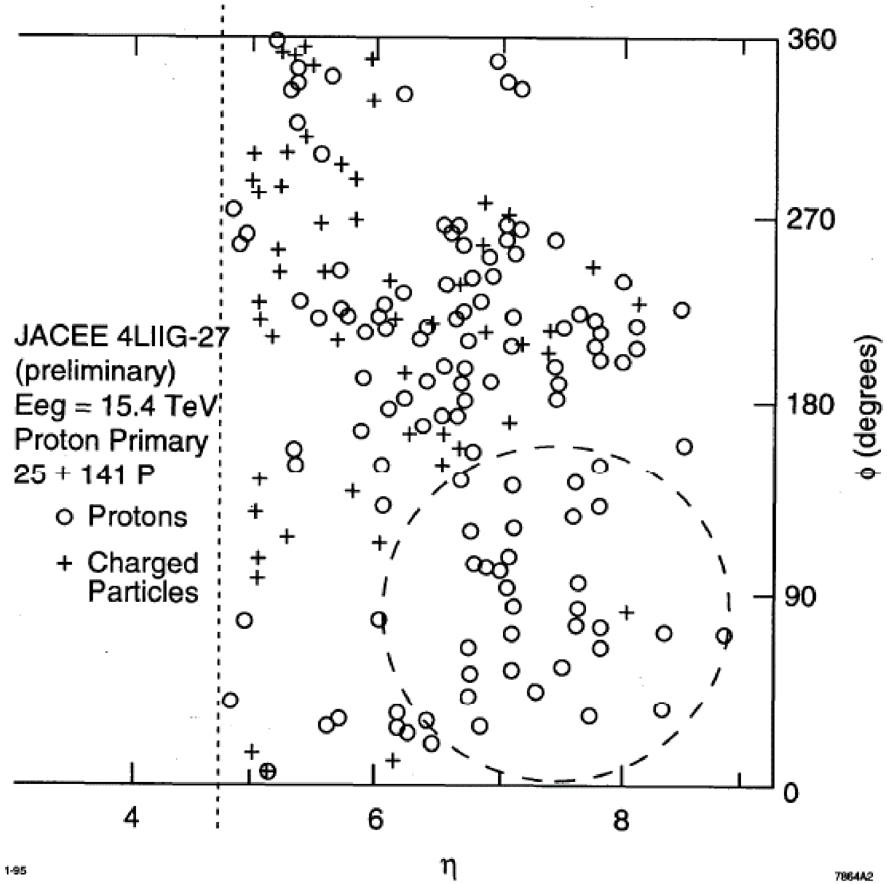


Nσ=0.5 j=2



High energy cosmic ray experiment and PHENIX

Can DCC scenario explain these events ?

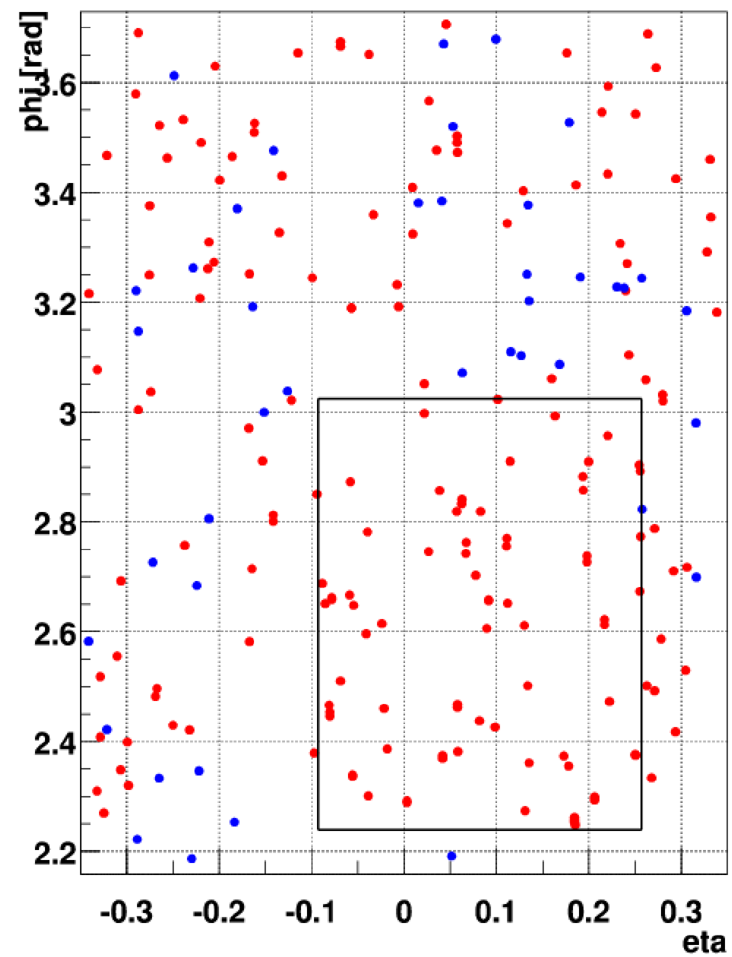


○: Photon

+ : Charged Particle

J. J. Lord and J. Iwai. Int. Conference on High Energy Physics, TX, 1992

PHENIX 7.24 standard deviation



● Charged track

● Photon cluster

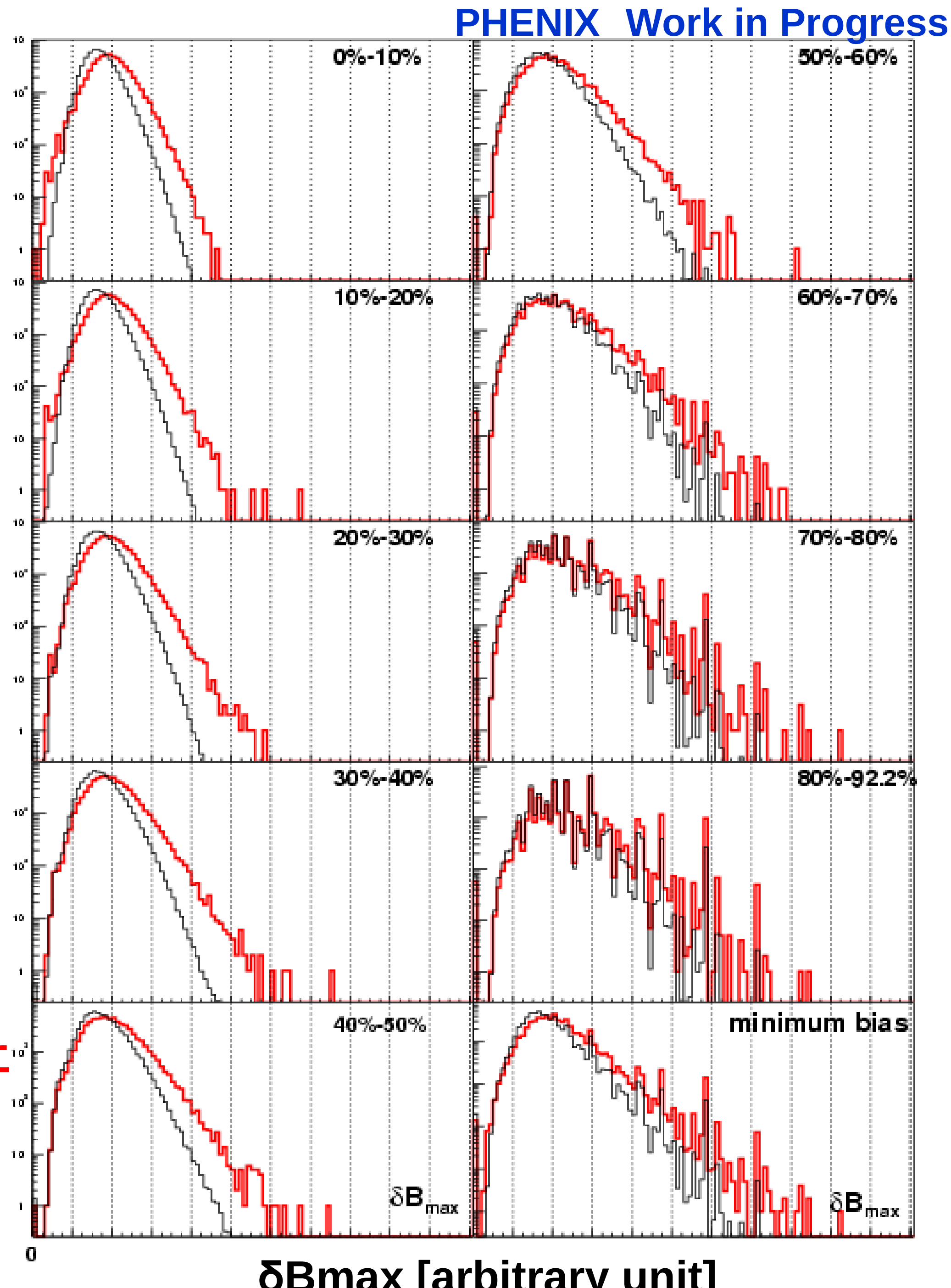
Maximum differential balance distributions

- **δBmax distribution**
  - black : binomial sample, 100 times larger statistics than real data obtained by hit map
  - red : data

明らかな離れ孤島は見つからず。

わざわざ大げさな探索しなくたって、
そもそもベースラインの分布は、
二項分布とは明らかに異なる。

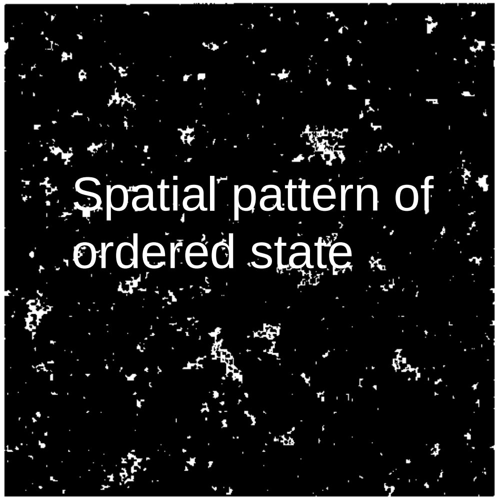
荷電πのみを使用して、熱・統計力学的に
分布を議論するほうが生産的。

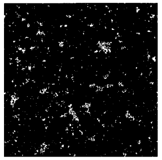


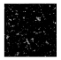
What is the critical behavior ?

Scale transformation

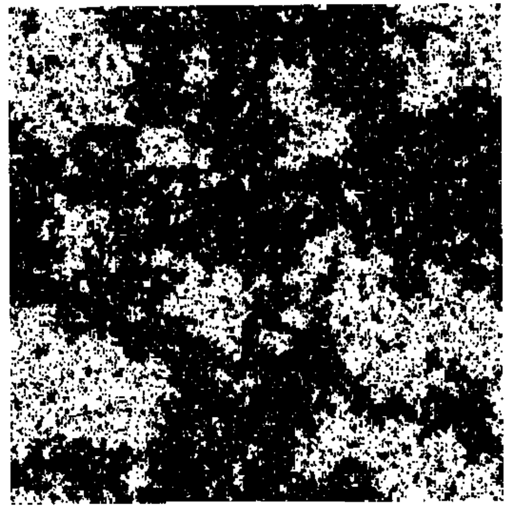
Ordered $T=0.995T_c$

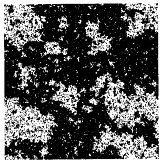


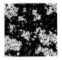


Black 

Critical $T=T_c$

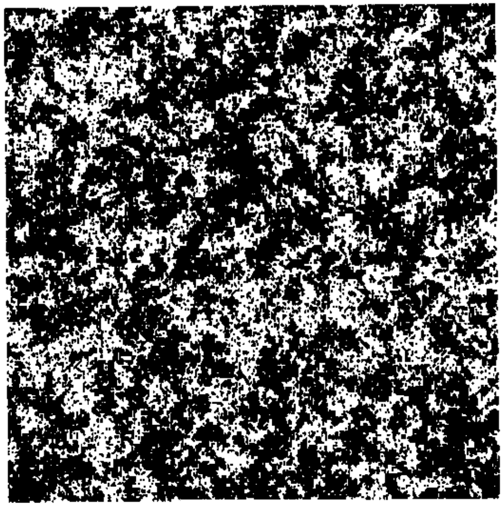


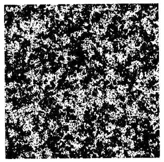


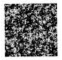
Black & White 

Various sizes
from small to large

Disordered $T=1.05T_c$



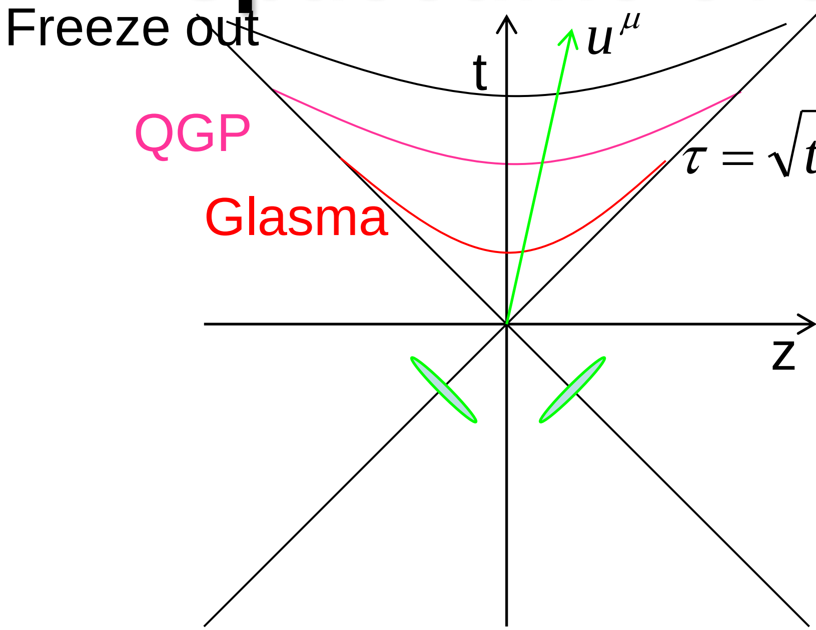


Gray 

Search for a transition
of the correlation size from T>Tc to T=Tc

A simulation based on two dimensional Ising model
from ISBN4-563-02435-X C3342I

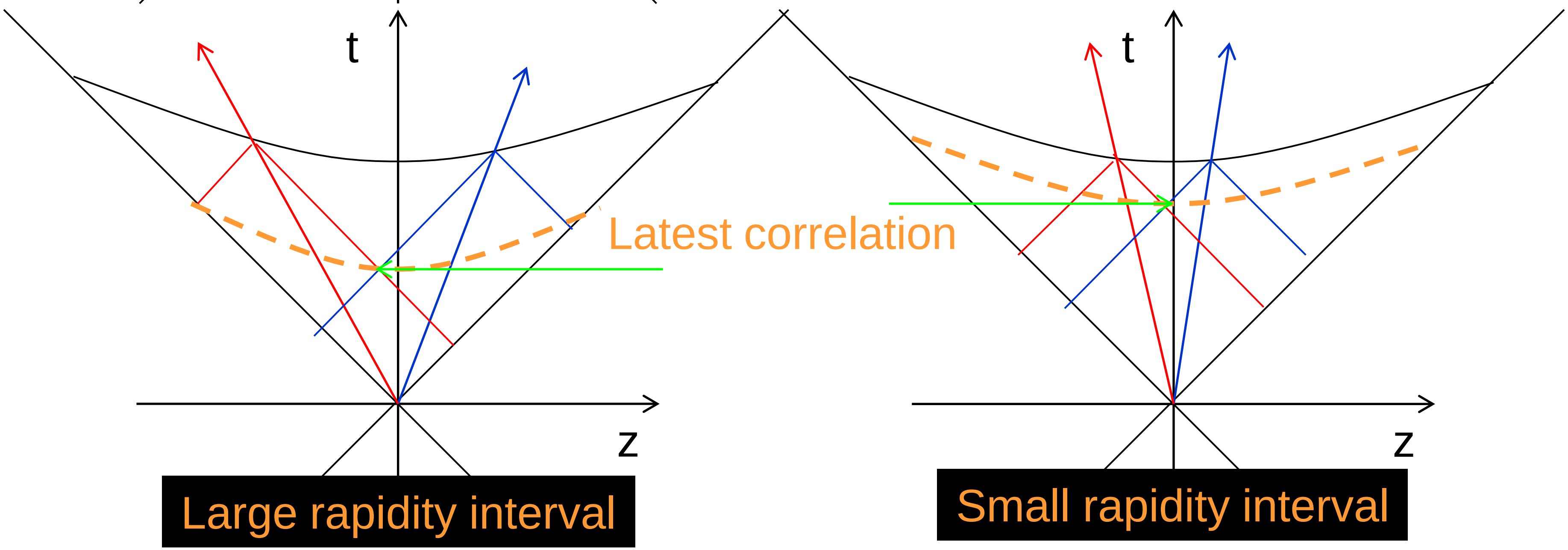
Spacetime evolution and Causality



$$\tau = \sqrt{t^2 - z^2} \quad u^\mu = (t/\tau, 0, 0, z/\tau) = (\cosh y, 0, 0, \sinh y)$$

$$y = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right) = \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)$$

$y \to \beta \quad \beta << 1$ (rapidity)

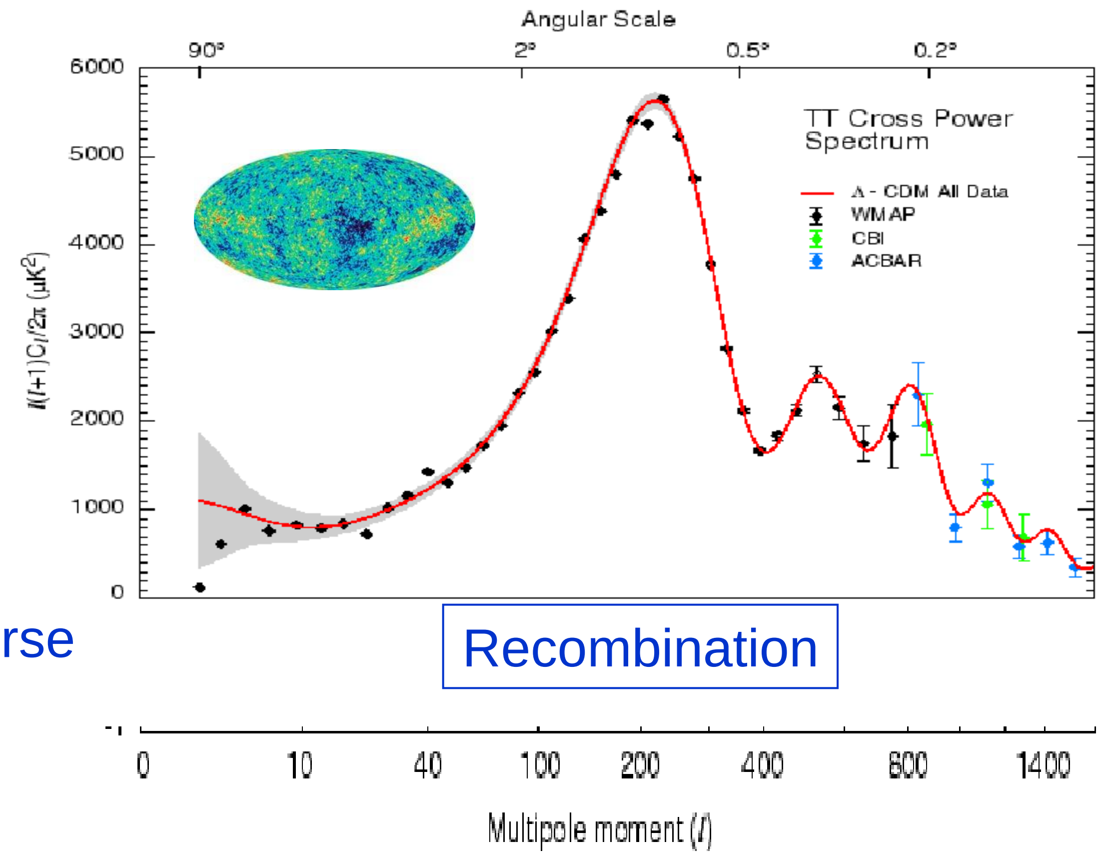


Analogue to the universe evolution

Long range rapidity correlation

QCD phase transition (Quark recombination)

Shorter range rapidity correlation



Early universe

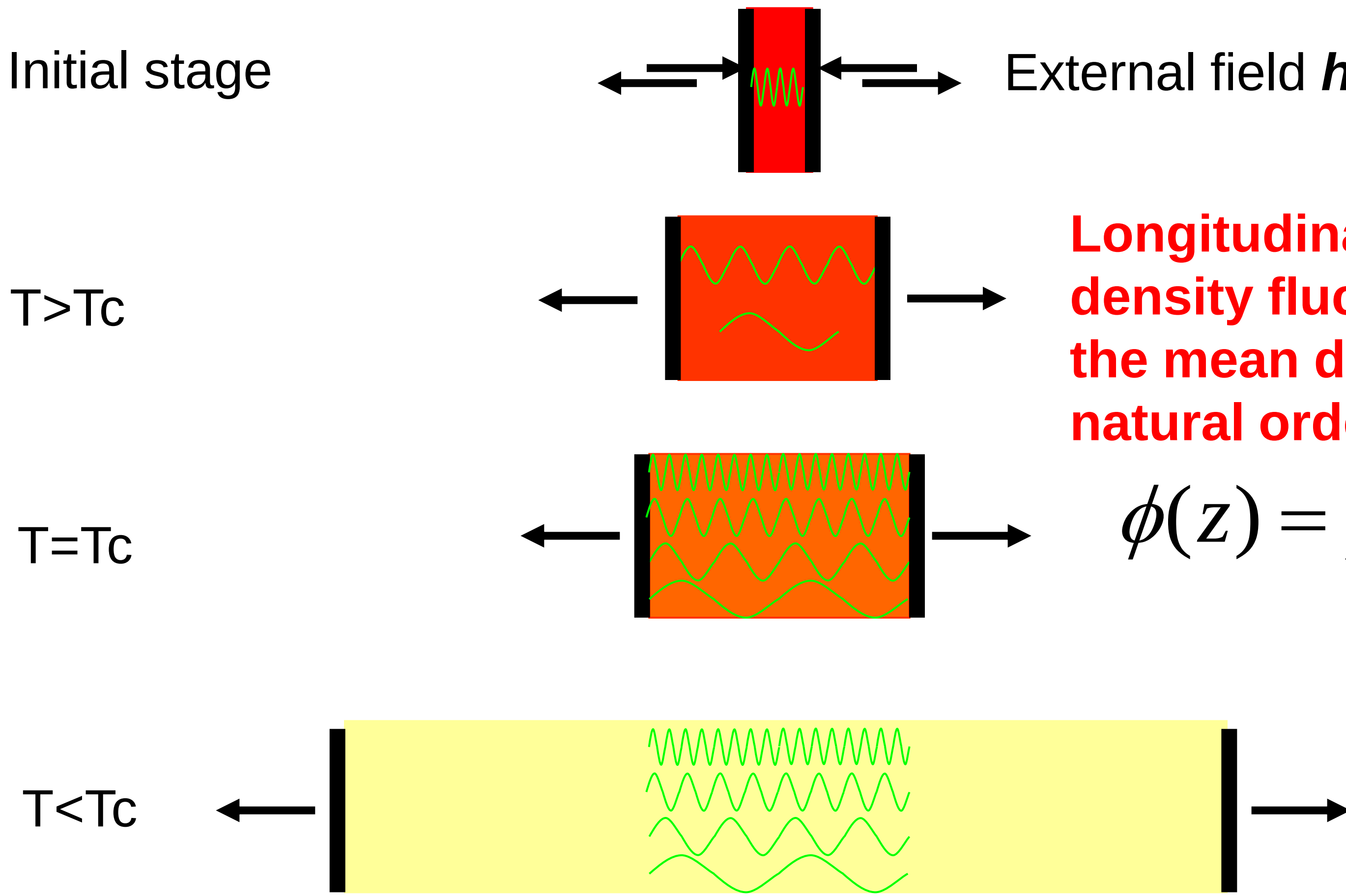
Recombination

Present universe

The Microwave Sky image from the WMAP Mission

http://map.gsfc.nasa.gov/m_mm.html

A picture of expanding medium in early stage



Longitudinal field density fluctuations from the mean density is a natural order parameter

$$\phi(z) = \rho(z) - \langle \rho \rangle$$

We may expect freeze of initially embedded fluctuation due to rapid dilution of medium in the longitudinal direction

Density-density correlation in longitudinal space

Longitudinal space coordinate z can be transformed into rapidity coordinate in each proper frame of sub element characterized by a formation time τ at which dominant density fluctuations are embedded.

$$z = \tau \sinh(y)$$

$$t = \tau \cosh(y)$$

$$dz = \tau \cosh(y) dy$$

Due to relatively rapid expansion in y, analysis in y would have an advantage to extract initial fluctuations compared to analysis in transverse plane in high energy collision.

$$g(T,\phi,h) - g_0 = \int_{\delta y} dy \int_{S\perp} d^2 x_\perp$$

$$\left[\frac{1}{2\tau^2 \cosh(y)} \left(\frac{\partial \phi}{\partial y} \right)^2 + \cosh(y) \left(\frac{1}{2} (\nabla_\perp \phi)^2 + U(\phi) \right) \right]$$

In narrow midrapidity region like PHENIX, cosh(y)~1 and y~η.

Direct observable for Tc determination

GL free energy density *g* with $\phi \sim 0$ from high temperature side is insensitive to transition order, but it can be sensitive to Tc

$$g(T,\phi,h) = g_0 - \underbrace{\frac{1}{2}A(T)(\nabla\phi)^2}_{\text{spatial correlation}} + \underbrace{\frac{1}{2}a(T)\phi^2}_{} + \cancel{\frac{1}{4}b\phi^4 + \frac{1}{6}c\phi^6} \cdots - h\phi$$

spatial correlation

ϕ disappears at Tc → $a(T) = a_0(T - T_c)$

Fourier analysis on

$G_2(y) = \langle\phi(0)\phi(y)\rangle$

$$\left\langle |\phi_k|^2 \right\rangle = Y \int G_2(y) e^{-ik(y)} dy$$

$$\left\langle |\phi_k|^2 \right\rangle = \frac{NT}{Y} \frac{1}{a(T) + A(T)k^2}$$

Susceptibility

$$\chi_k = \frac{\partial \phi_k}{\partial h} \propto \left(\frac{\partial^2 (g - g_0)}{\partial \phi^2{}_k} \right)^{-1} = \frac{1}{a_0(T - T_c)(1 + k^2\xi^2)}$$

Susceptibility in long wavelength limit

$$\chi_{k=0} = \frac{1}{a_0(T - T_c)} \propto \frac{\xi}{T} G_2(0)$$

1-D two point correlation function

$$G_2(y) = \frac{NT}{2Y^2 A(T)} \xi(T) e^{-|y|/\xi(T)}$$

Correlation length

$$\xi(T)^2 \equiv \frac{A(T)}{a_0(T - T_c)}$$

Product between correlation length and amplitude can also be a good indicator for T~Tc

Strategy to find phase transition

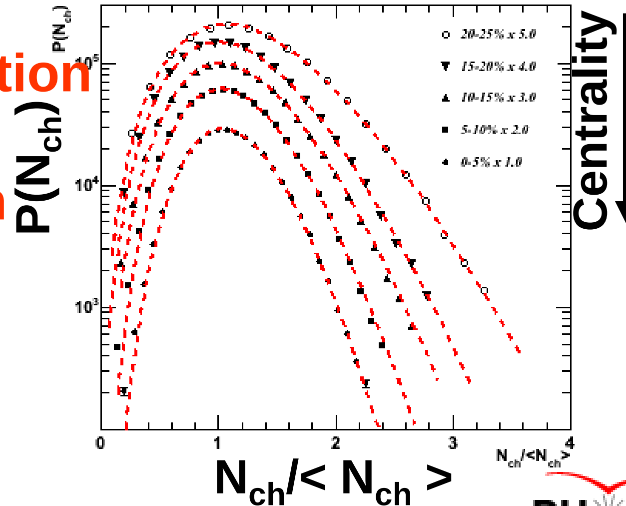
Step1.
Search for increase of correlation length and susceptibility (amplitude x correlation length) determined by exponential form in T>Tc→T~Tc

Step2.
Search for transition of two point correlation from exponential to power law form which needs higher order terms in the free energy density. This would be a stronger indication of T=Tc.

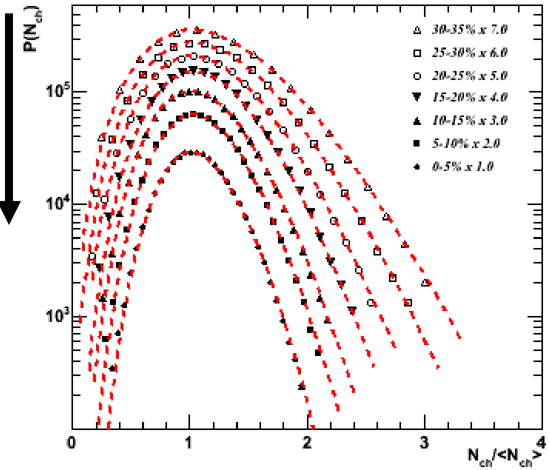
Density measurement: inclusive $dN_{ch}/d\eta$

Negative Binomial Distribution (NBD) perfectly describes multiplicities in all collision systems and centralities at RHIC.

Cu+Cu@62.4GeV

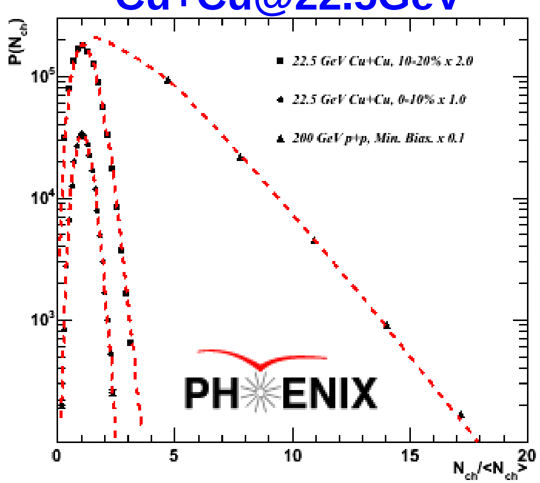


Cu+Cu@200GeV

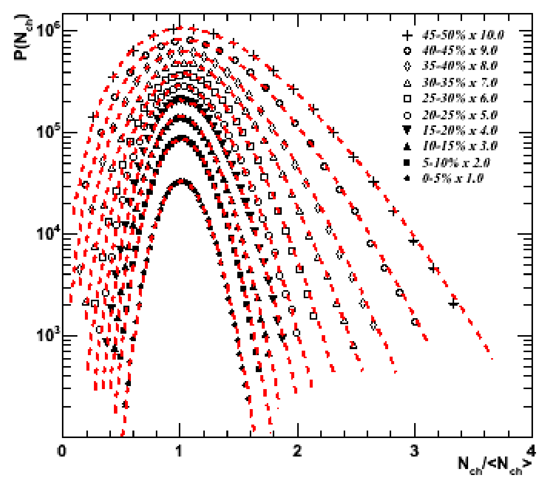


PHENIX

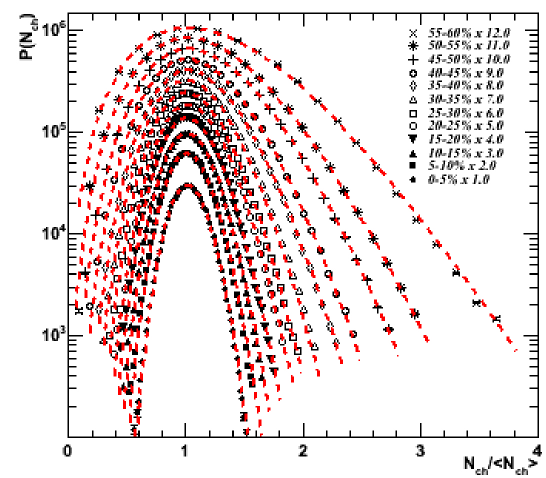
p+p@200GeV
Cu+Cu@22.5GeV



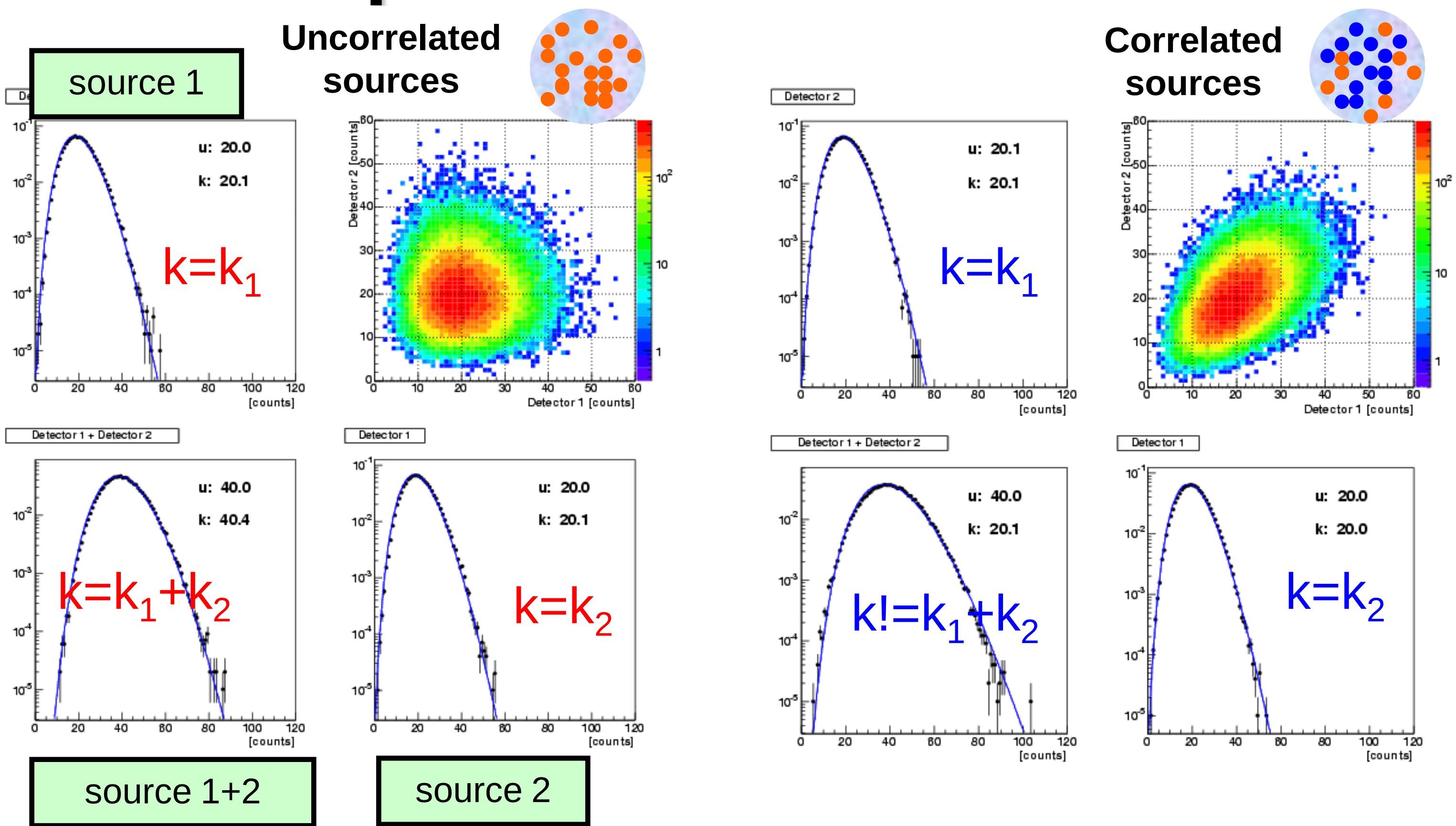
Au+Au@62.4GeV



Au+Au@200GeV



Two point correlation via NBD



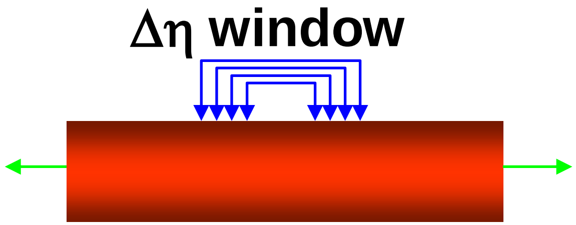
NBD $$P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left(\frac{\mu/k}{1+\mu/k} \right)^n \frac{1}{(1+\mu/k)^k}$$

k=1 Bose-Einstein
k=∞ Poisson

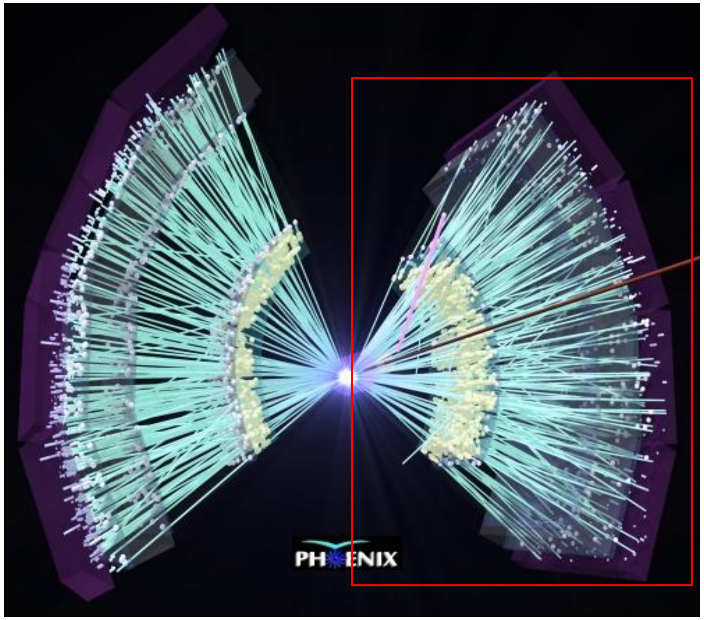
$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \boxed{\frac{1}{k}} \qquad \mu \equiv \langle n \rangle$$

1/k corresponds to integral of two point correlation

Differential multiplicity measurements



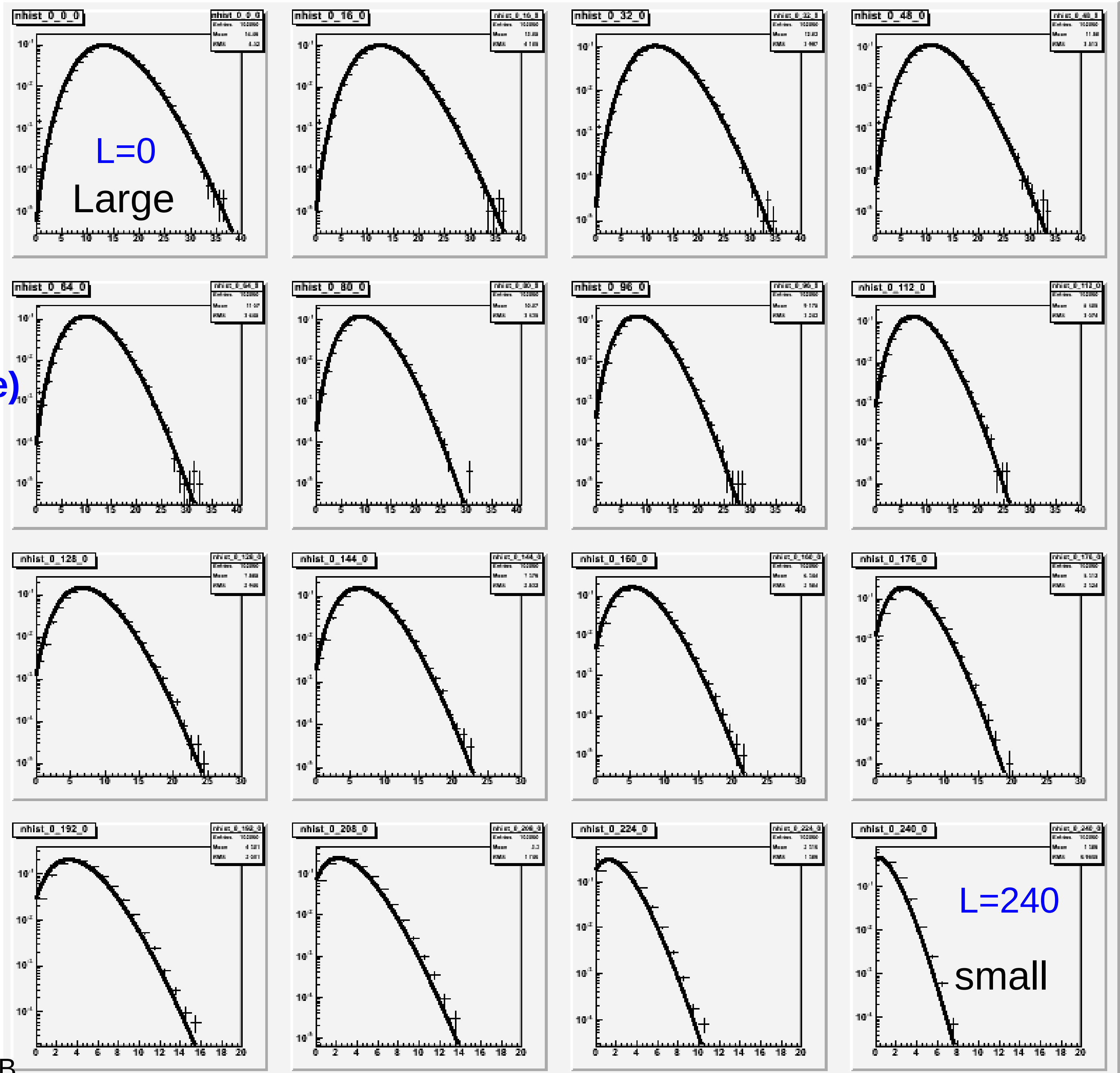
Δη<0.7 integrated over Δφ<π/2 and pT>0.1GeV



Zero magnetic field to enhance low pt statistics per collision event.

NBD fits at each window size in CuCu@200

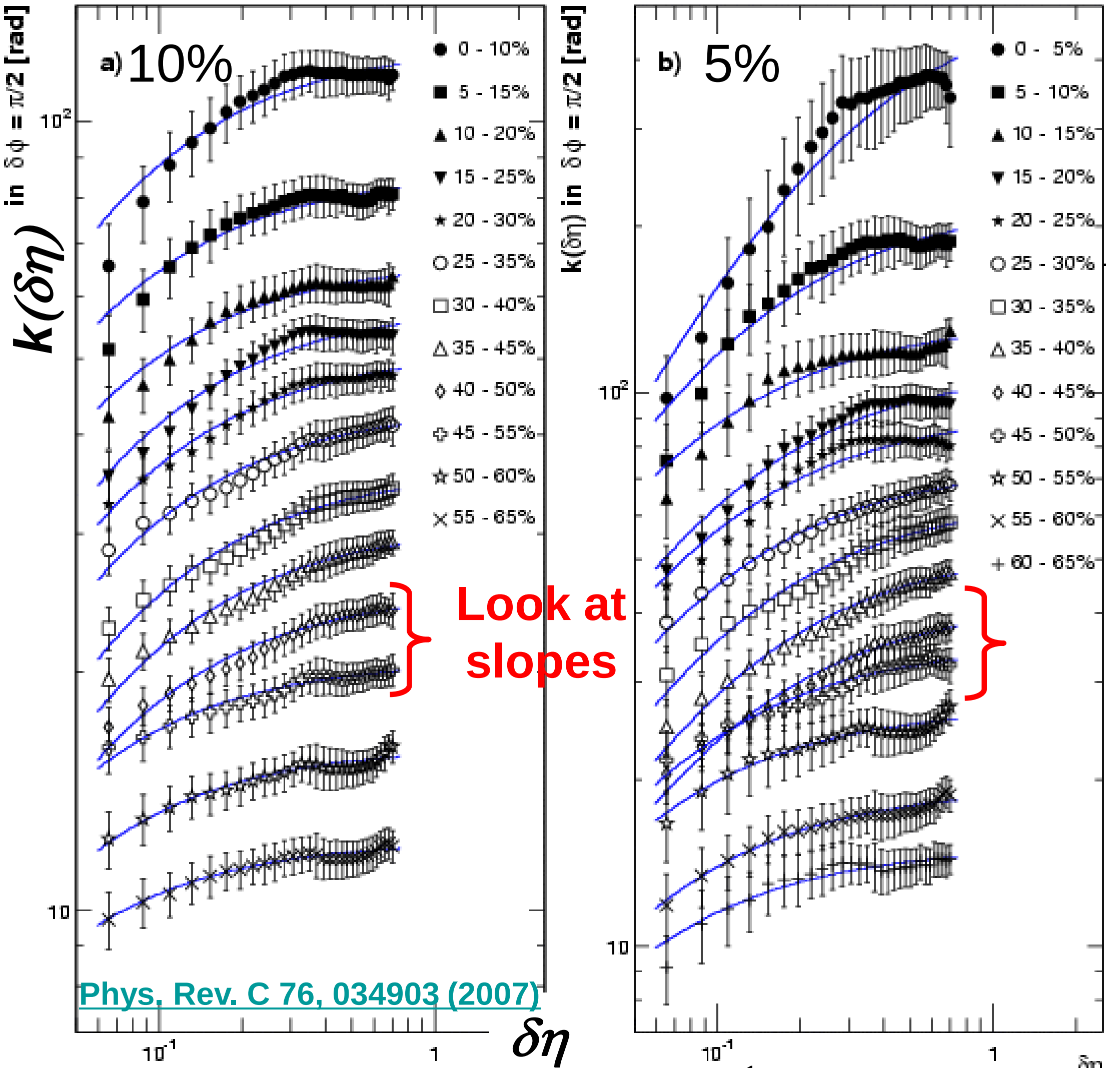
Level (window size)
$L=2^8(1-\delta\eta/\Delta\eta_{PHENIX})$



16 fit examples in most left edge in top 10% events out of $2^8/2*(1+2^8)$ times NBD fits

Extraction of αξ product

Fit with approximated functional form



Approximated functional form

$$k(\delta\eta) = \frac{1}{2\alpha\xi/\delta\eta + \beta} \quad (\xi << \delta\eta)$$

Parametrization of two particle correlation

$$C_2(\eta_1,\eta_2) \equiv \rho_2(\eta_1,\eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$$

$$\frac{C_2(\eta_1,\eta_2)}{\bar{\rho}_1^2} = \alpha e^{-\delta\eta/\xi} + \beta$$

β absorbs rapidity independent bias: Npart fluctuation, reaction plane rotation, and v2

Exact relation with NBD *k*

$$k^{-1}(\delta\eta) = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} - 1$$

$$= \frac{\int_0^{\delta\eta}\int_0^{\delta\eta} C_2(\eta_1,\eta_2)d\eta_1 d\eta_2}{\delta\eta^2 \bar{\rho}_1^2}$$

$$= \frac{2\alpha\xi^2(\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}{\delta\eta^2} + \beta$$

Correlation functions and correlation length

Used in E802

$$C_2 = 1 + R(0,0)e^{-|y_1 - y_2|/\xi}$$

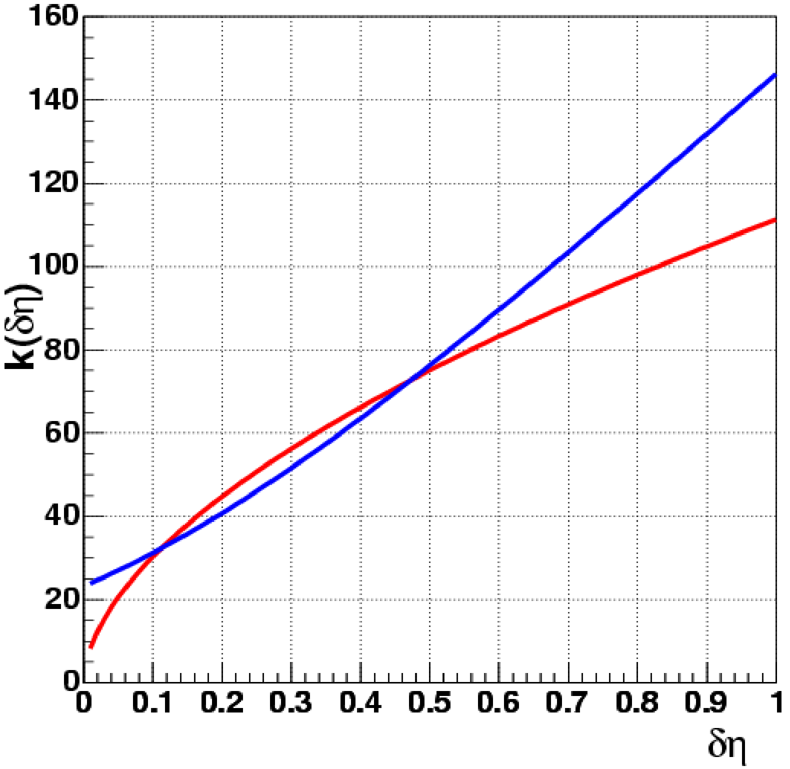
$$k(\delta\eta) = \frac{1}{R_0} \frac{\delta\eta / 2\xi}{[1 - (\xi / \delta\eta)(1 - e^{-\delta\eta/\xi})]}$$

General correlation function

$$C_2 = 1 + \frac{R_0}{|y_1 - y_2|^{\alpha}} e^{-|y_1 - y_2|/\xi}$$

$$k(\delta\eta) = \frac{\delta\eta}{\int^{\delta\eta} \frac{R_0}{y^{\alpha}} e^{-y/\xi} dy}$$

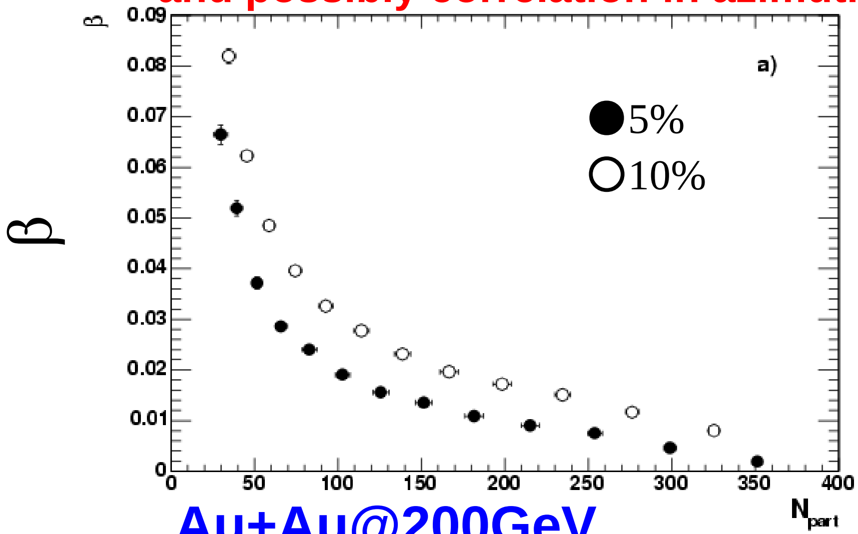
Using arbitrary R_0, **ξ** and **α.**



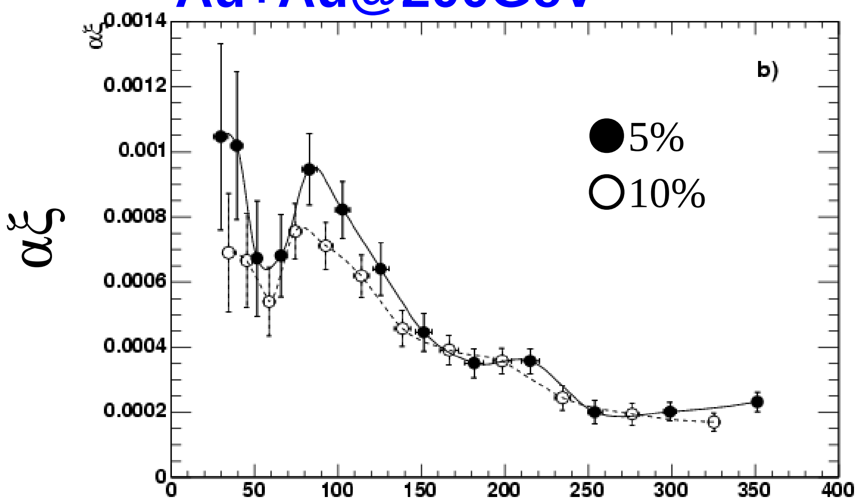
ξ : correlation length, **α :** critical exponent

αξ, β vs. Npart

Dominantly Npart fluctuations
and possibly correlation in azimuth



Au+Au@200GeV



Phys. Rev. C 76, 034903 (2007)

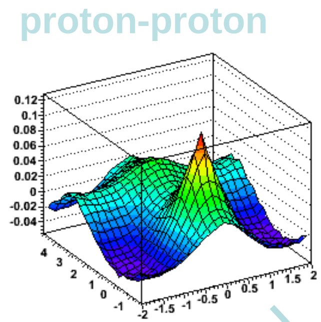
Npart

β is systematically shift to lower values as the centrality bin width becomes smaller from 10% to 5%. This is understood as fluctuations of Npart for given bin widths

αξ product, which is monotonically related with $\chi_{k=0}$ indicates the non-monotonic behavior around Npart ~ 90.

$$\alpha\xi = \chi_{k=0} T / \bar{\rho}_1^{\,2} \propto \bar{\rho}_1^{\,-2} \frac{T}{|T - T_C|}$$

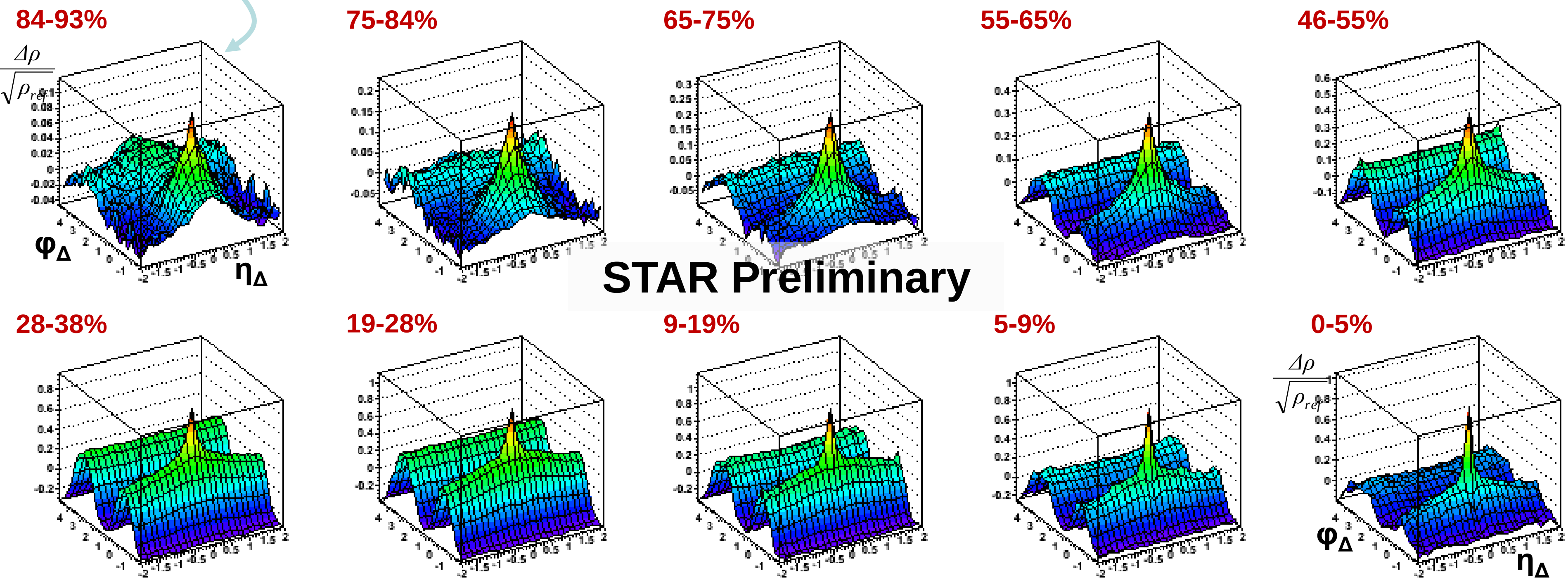
Significance with Power + Gaussian:
3.98 σ (5%), 3.21 σ (10%)
Significance with Line + Gaussian:
1.24 σ (5%), 1.69 σ (10%)



How about STAR?

Analyzed 1.2M minbias 200 GeV Au+Au events, and 13M 62 GeV minbias events (not shown) Included all tracks with $p_T > 0.15$ GeV/c, $|\eta| < 1$, full φ

note: 38-46% not shown



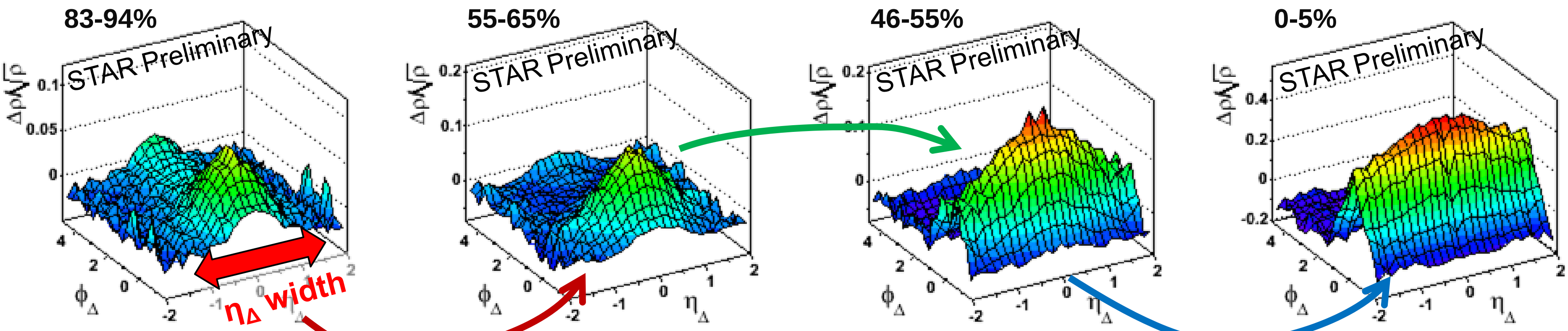
We see the evolution of correlation structures from peripheral to central Au+Au

Slide from M. Daugherity, STAR Collaboration presented at QM08

Transition

Does the transition from narrow to broad η_Δ occur quickly or slowly?

data - fit (except same-side peak)



Shape changes little from peripheral to the transition

Large change within ~10% centrality

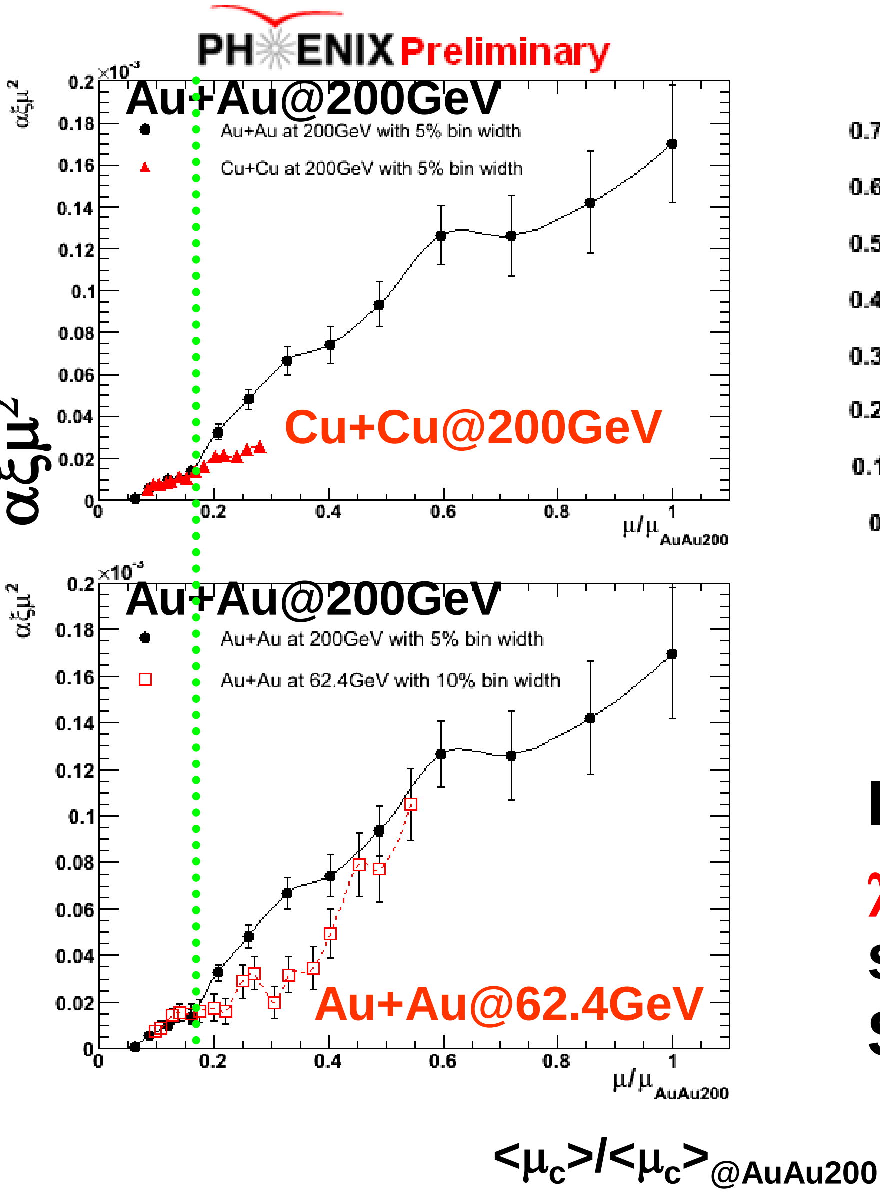
Smaller change from transition to most central

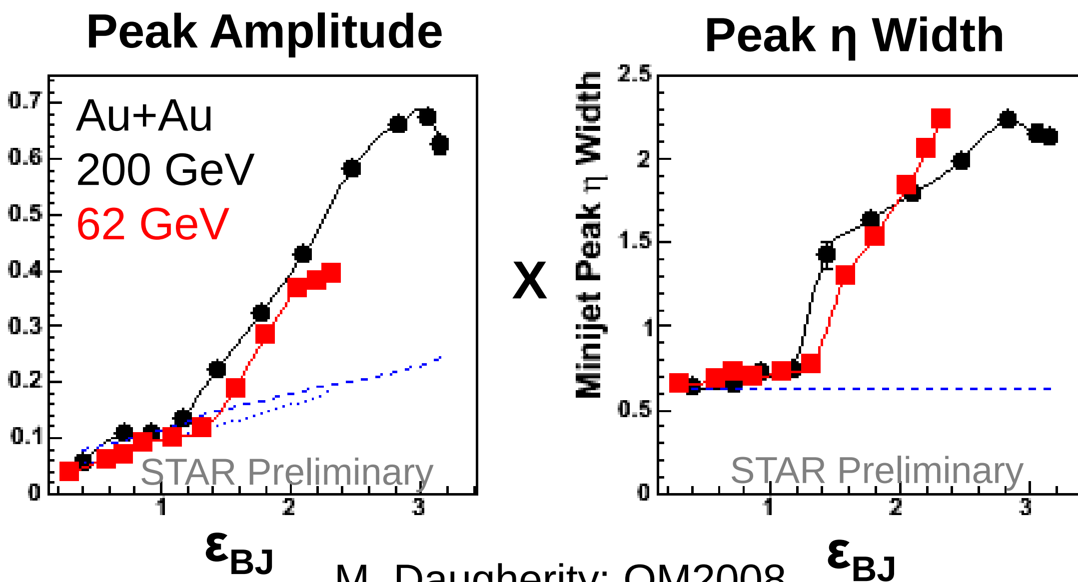
Low-p_T manifestation of the “ridge”

The transition occurs quickly

Slide from M. Daugherity, STAR Collaboration presented at QM08

Similarity to STAR mini jet results at low p_T



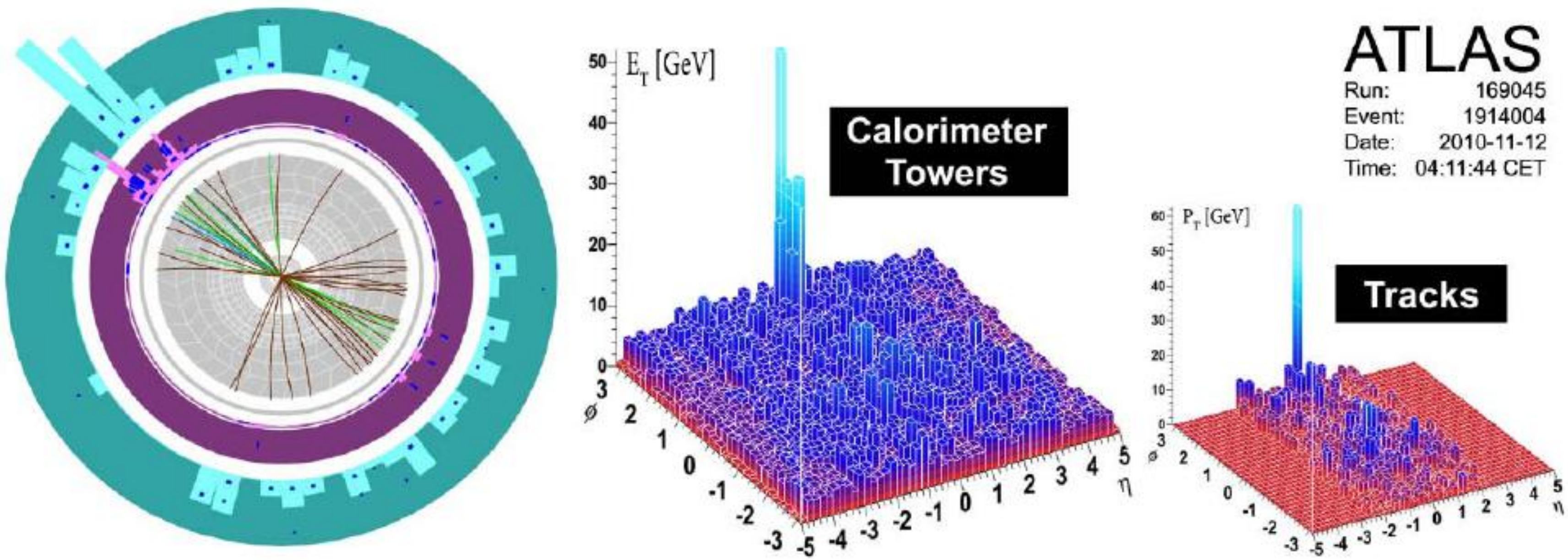


M. Daugherity: QM2008

Equivalent quantity;
$\chi T \propto \alpha\xi\mu^2 \propto$ amplitude x width
shows similar trends to what STAR sees.

1事象で片付く例

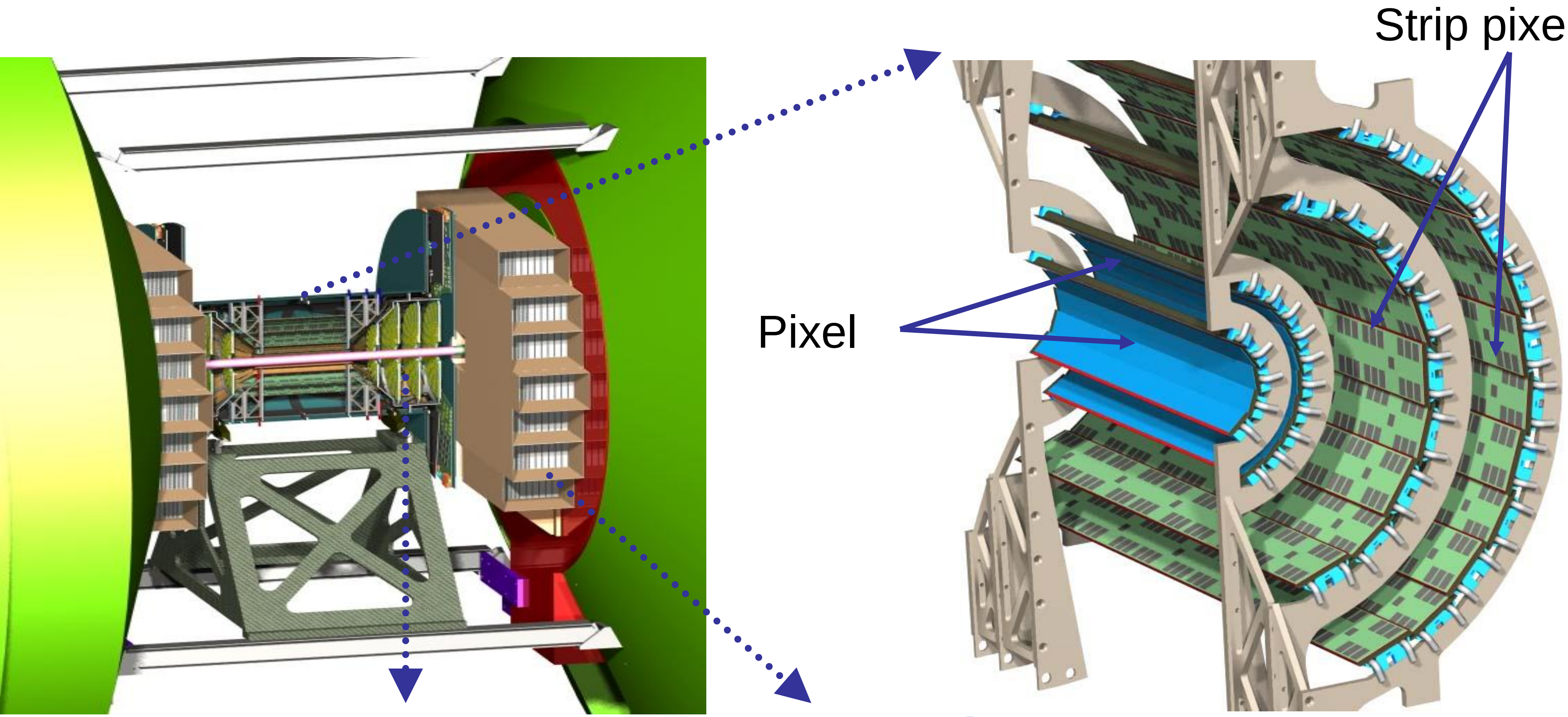
First observation



- Large energy imbalance between leading and subleading jet in central Pb+Pb collisions seen at the event by event basis.
- Quantification of the effect:
  - dijet asymmetry $A_J = \frac{E_{T1} - E_{T2}}{E_{T1} + E_{T2}}$ … quantifies the energy imbalance
  - dijet $\Delta\phi = |\phi_2 - \phi_1|$ … quantifies the azimuthal correlation

VTX, FVTX, and NCC for future runs

Central Vertex detector (VTX)



Nose Cone Calorimeter

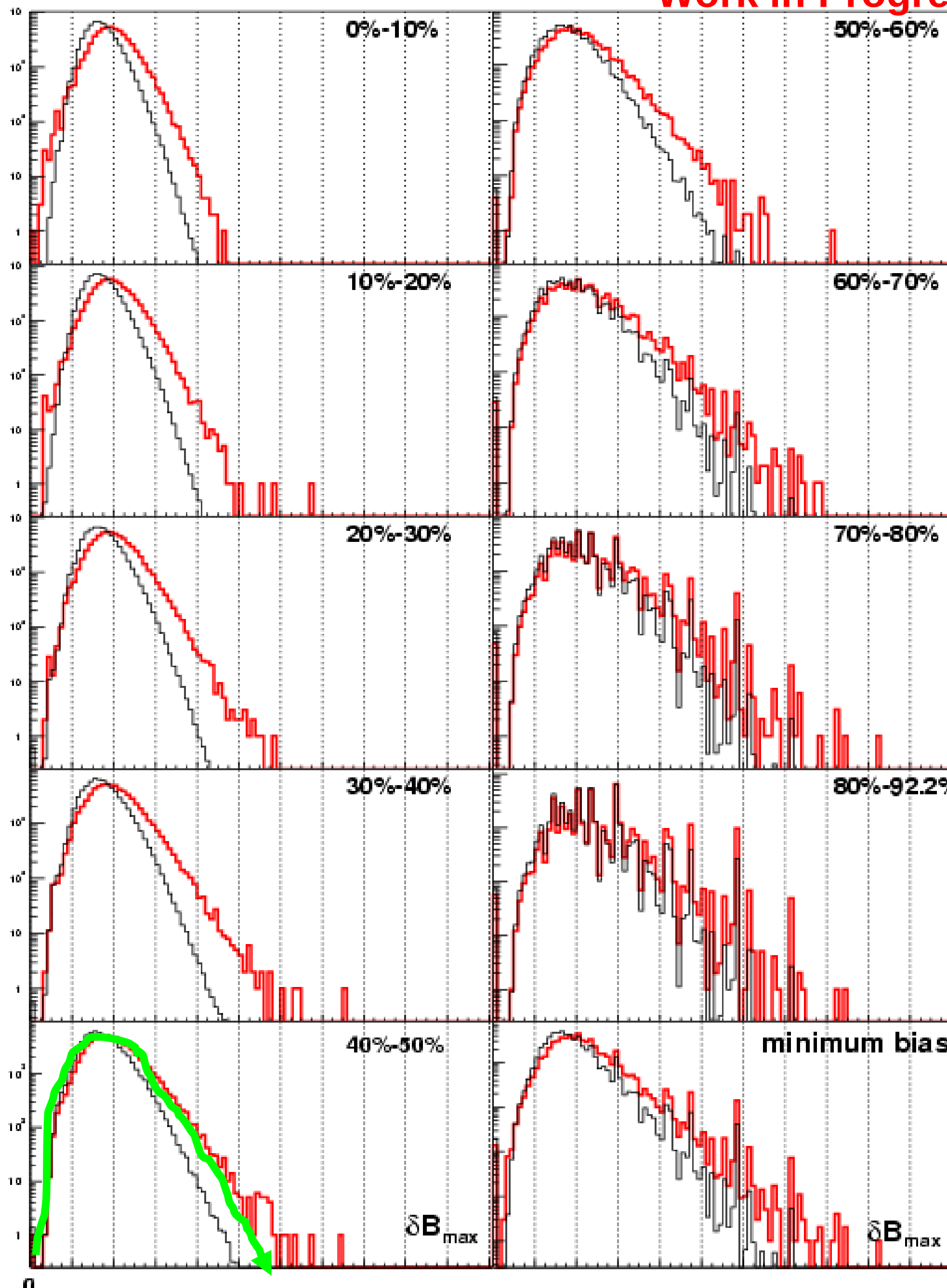
Forward VTX (FVTX) (NCC)

PHENIX can extend both rapidity and azimuthal coverage

Maximum differential balance distributions

- ベースラインは、もう押さえられているはずであるから、この解析に戻ることが可能。より大きな検出器アクセプタンスにおいて、より多くの統計を用いて、大きな値を取る異常な揺らぎを視覚的かつ定量的に議論することができる(かも)。

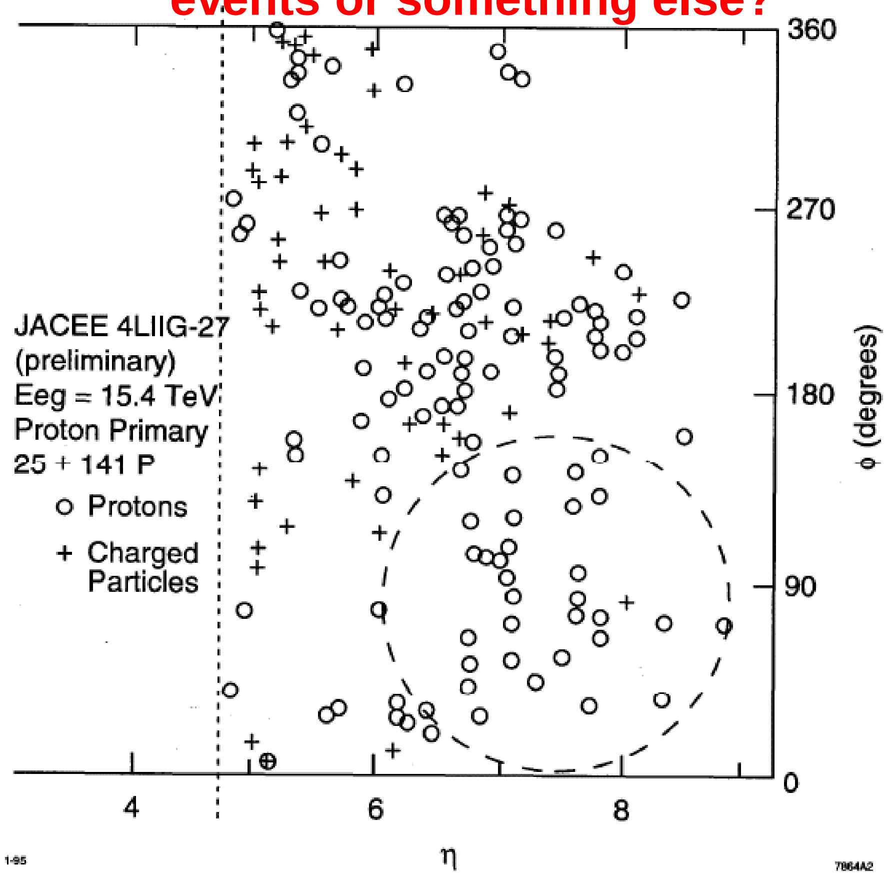
Work in Progress



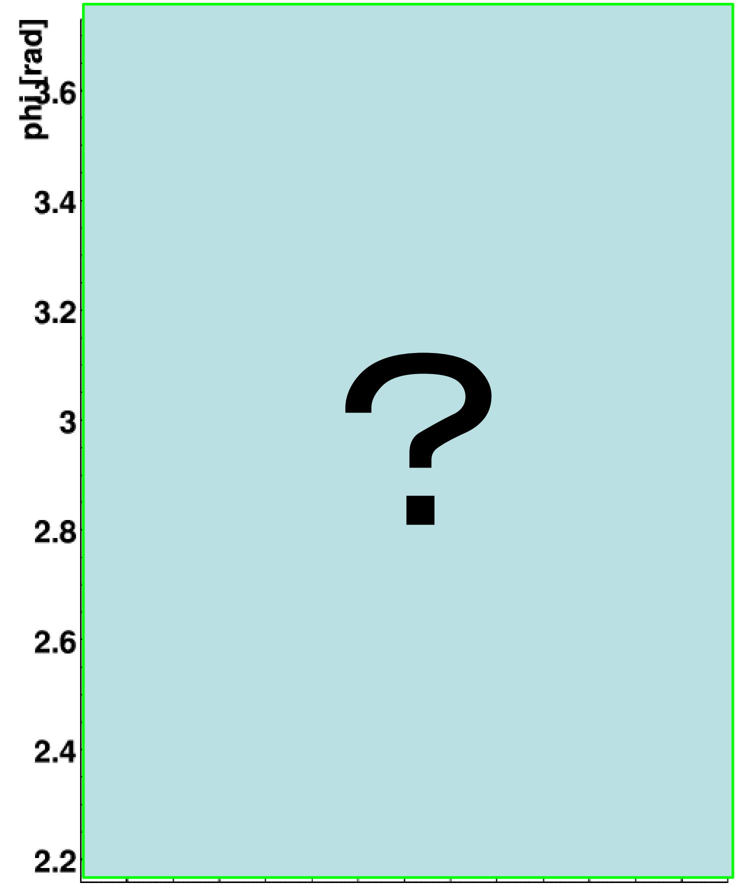
δBmax [arbitrary unit]

まとめ：　事象毎の揺らぎ解析への回帰

Can DCC scenario explain these events or something else?



New PHENIX



○: Photon

+ : Charged Particle

J. J. Lord and J. Iwai. Int. Conference on High Energy Physics, TX, 1992

- **Charged track**
- **Photon cluster**