エネルギー走査実験とバリオン数ゆらぎ

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Heavy Ion Pub, 2011/12/16, Osaka U.

Outline

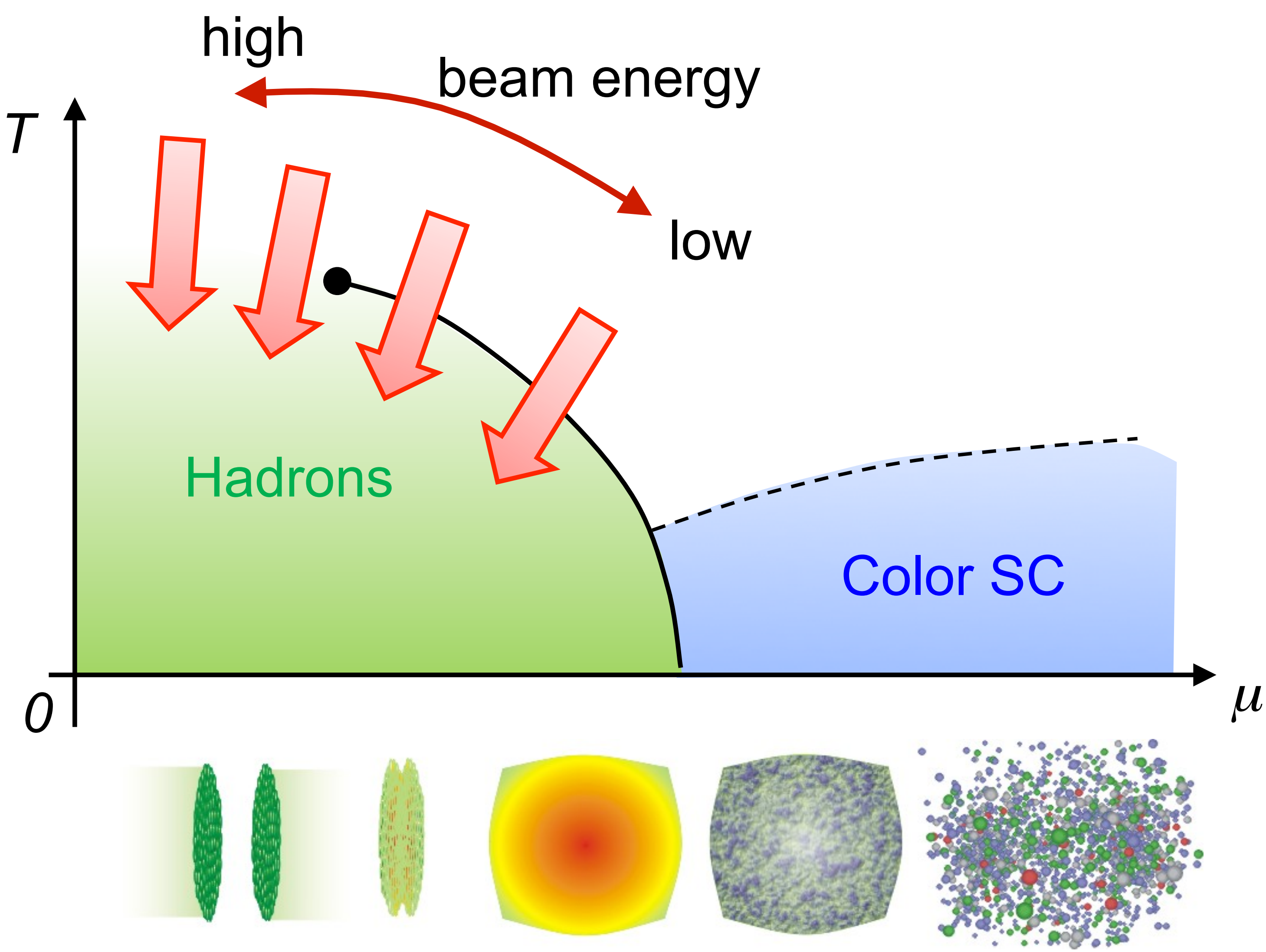
1. ゆらぎを用いたQCD相構造の探索

2. バリオンおよび陽子数ゆらぎの関係について

MK, Asakawa, arXiv:1107.2755

The authors thank stimulative discussions at the workshop "Frontiers of QCD Matter" held at Nagoya University, Japan, on June 8th, 2011, and H. Torii for the organization. This work is supported in part by Grants-in-Aid for Scientific Research by Monbu-Kagakusyo of Japan (No. 21740182 and 23540307).

Energy Scan Program @ RHIC



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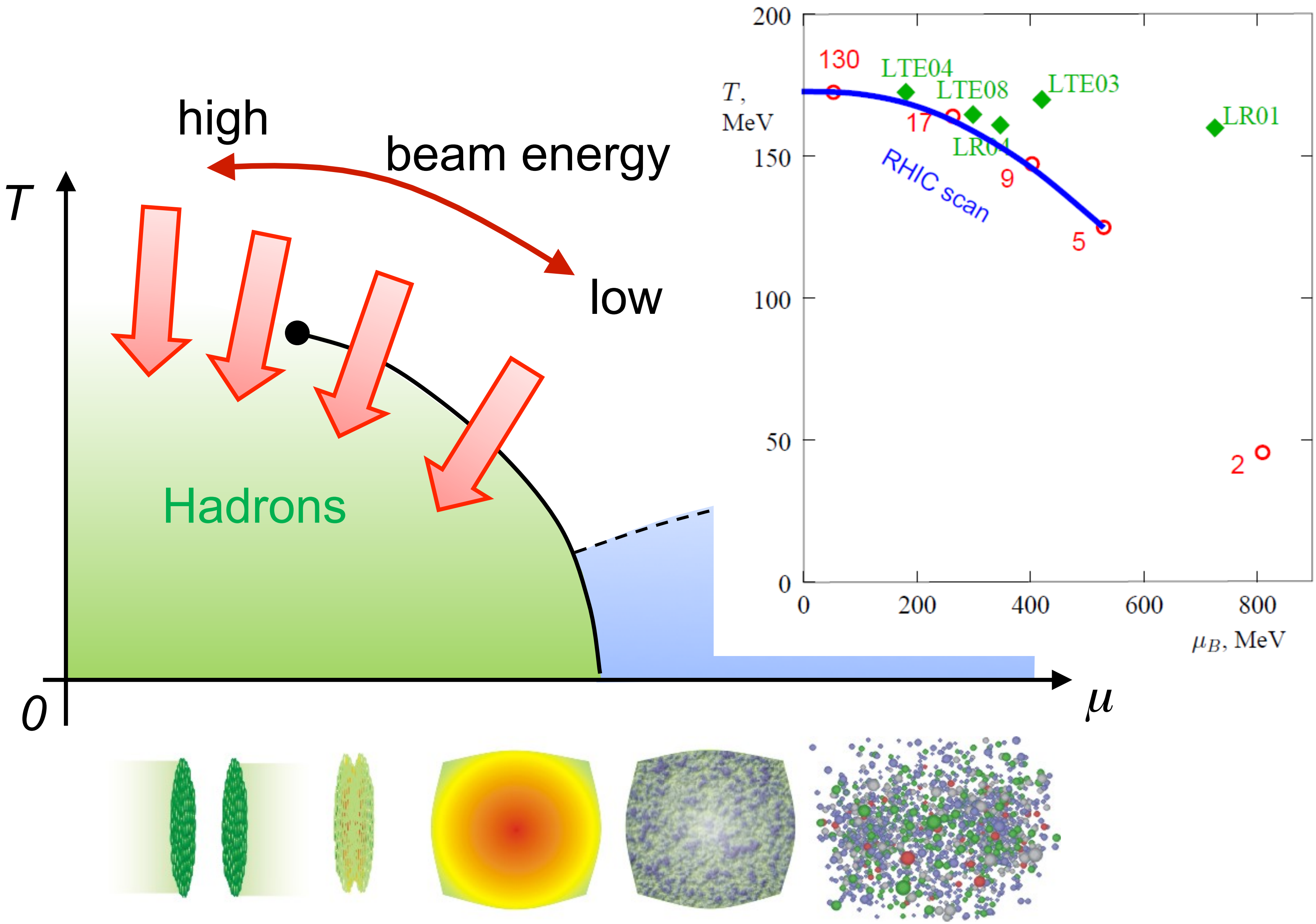
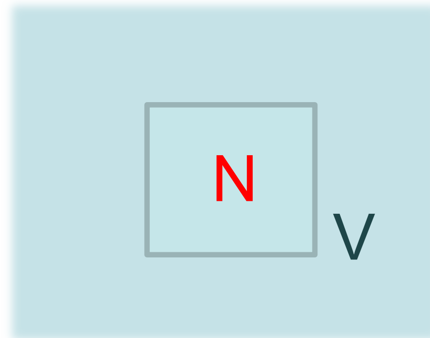
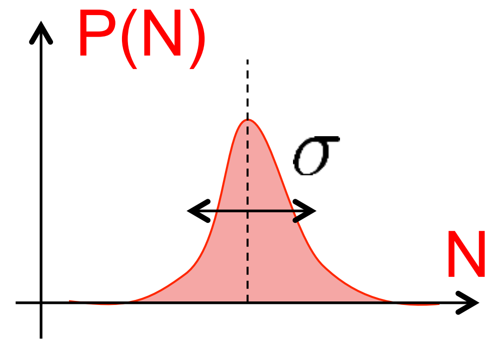


Fluctuations

平衡状態において、
物理量はゆらいでいる。

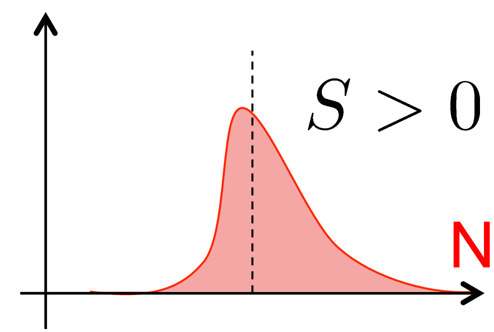




ゆらぎを特徴づける量

$$\delta N = N - \langle N \rangle$$

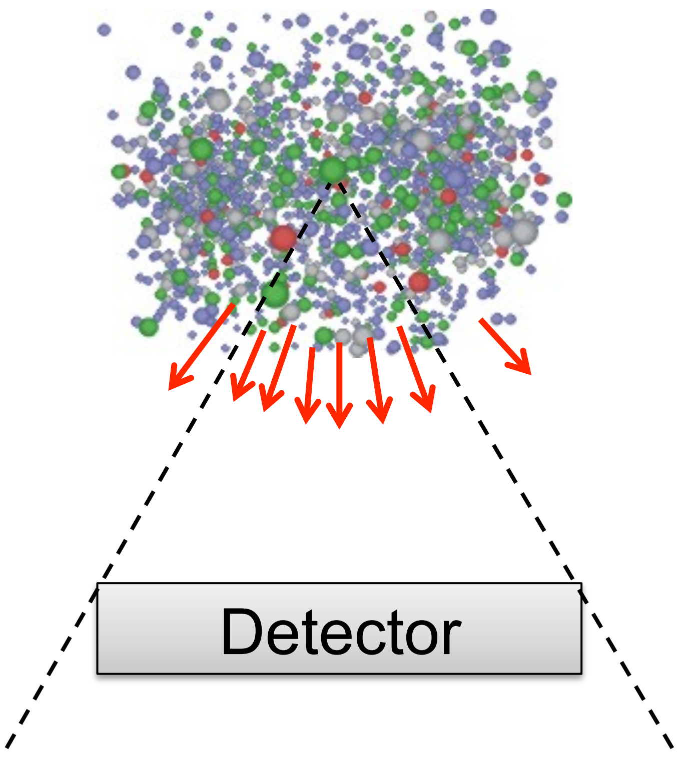
- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$

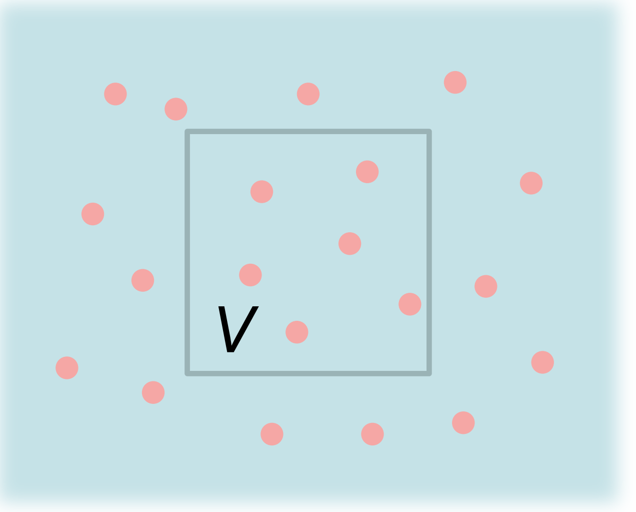


- Kurtosis: $\kappa = \dfrac{\chi_4}{\chi_2 \sigma^2}$ ⇐ $V\chi_4 = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2$
- And much higher…

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.





Variation of N_Q in a rapidity range is small for conserved charges.

Asakawa, et al., ’00; Jeon, Koch, ’00; Shuryak, Stephanov, ’02

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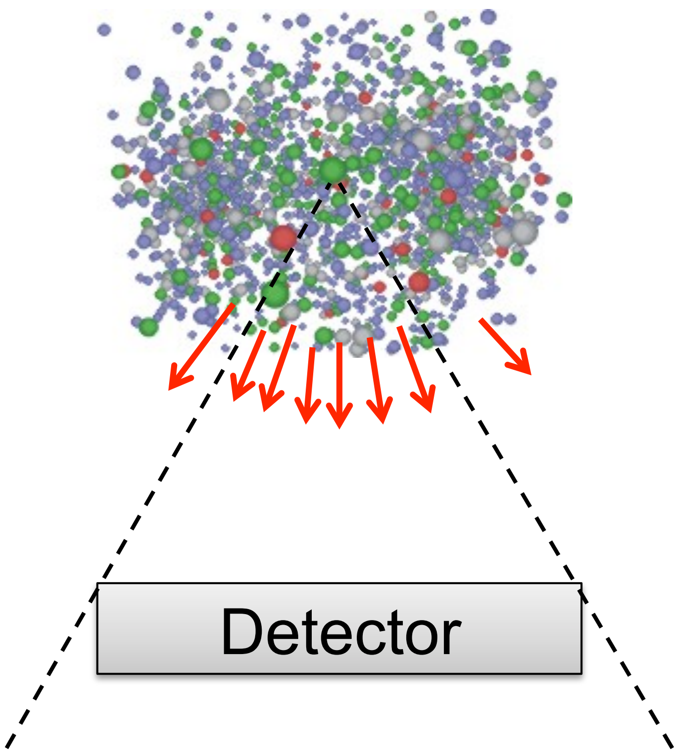
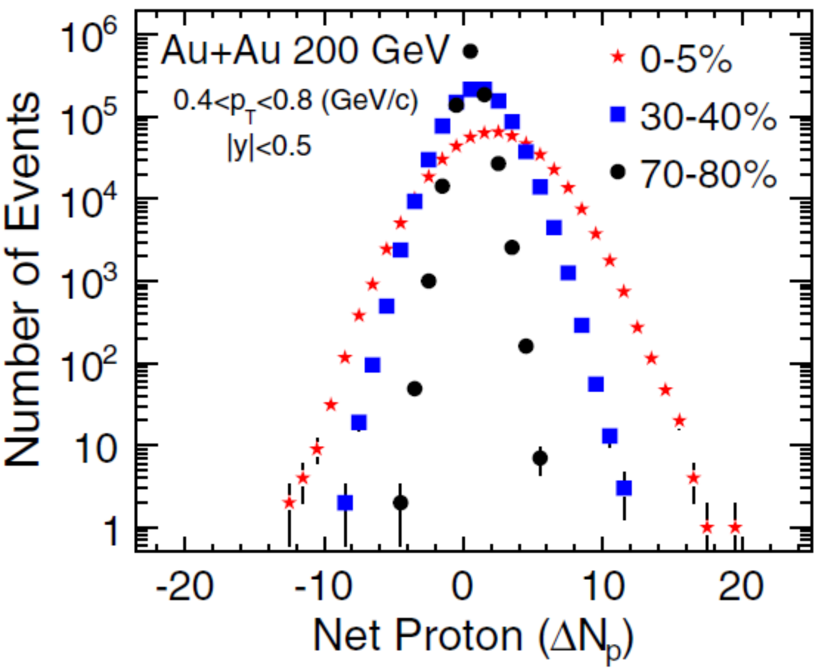


STAR, PRL**105** (2010)



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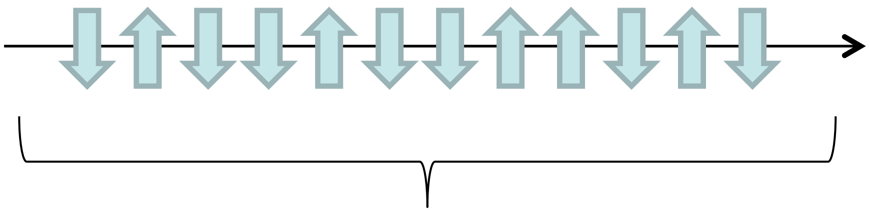
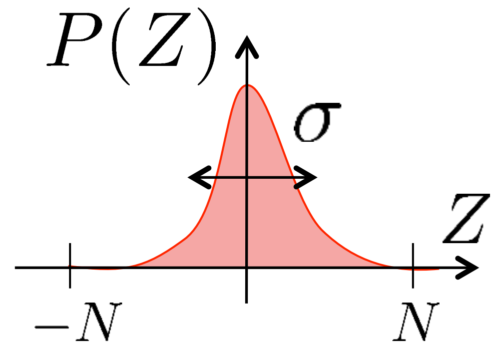
イジング模型とゆらぎ

$$Z = \sum z_i \qquad \sigma^2 = \langle \delta Z^2 \rangle$$

$T \gg T_c$



*N*個のスピン



$$\frac{\sigma}{N} = \frac{1}{\sqrt{N}}$$

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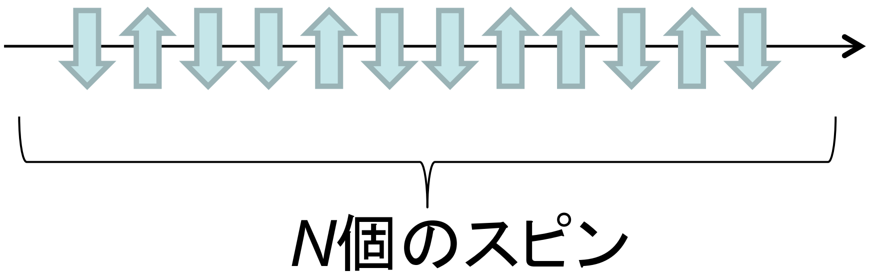

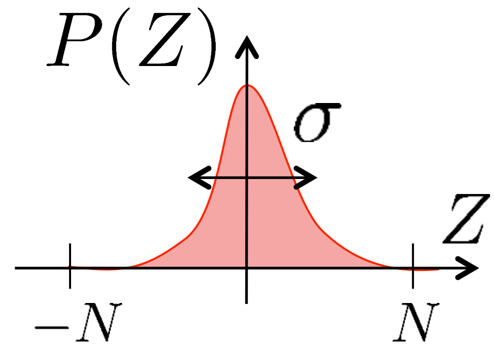

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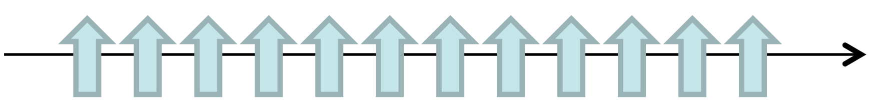

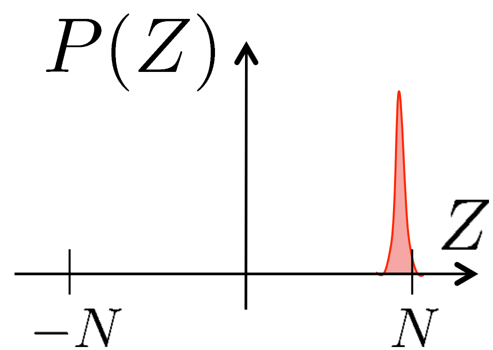

$$\frac{\sigma}{N} \simeq 0$$

イジング模型とゆらぎ

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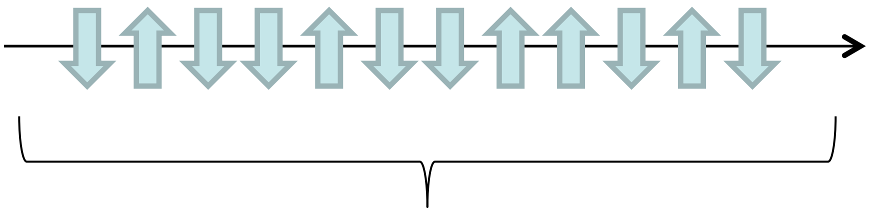
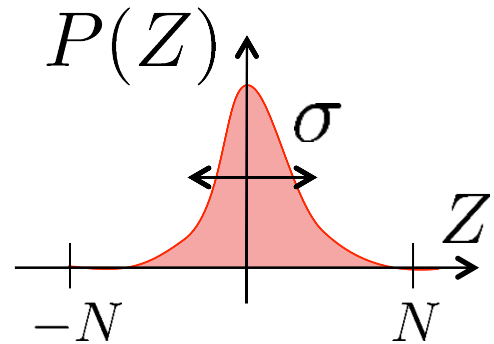


N個のスピン



$$\frac{\sigma}{N} = \frac{1}{\sqrt{N}}$$

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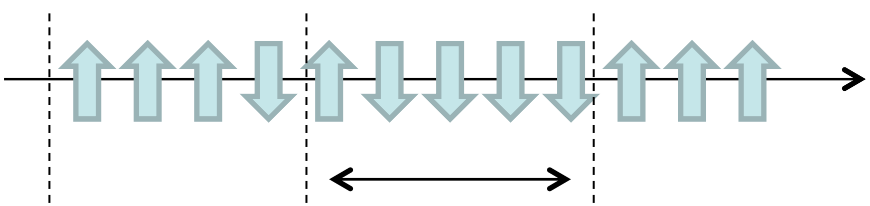


長さLで大体同じ向き

N/L個の独立なスピンの集団

$$\frac{\sigma}{N} = \sqrt{\frac{L}{N}}$$

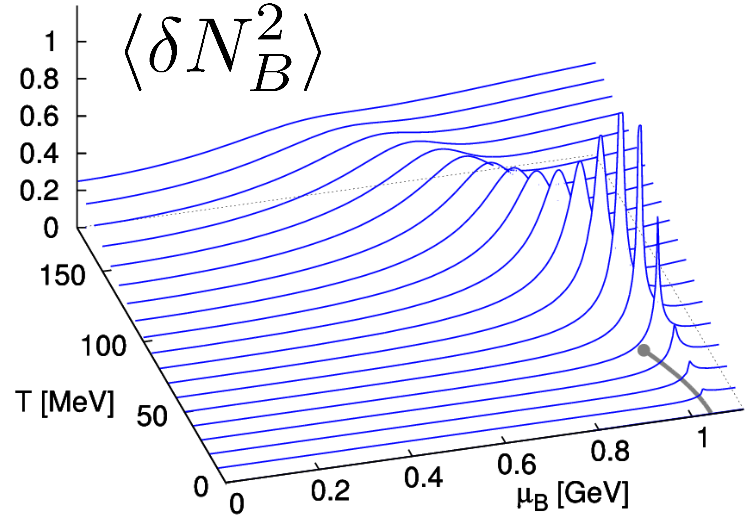
ゆらぎは$L^{d/2}$で増大し、臨界点で発散する。

Fluctuations at QCD Critical Point

Stephanov, Rajagopal, Shuryak '98,'99

- 2^{nd} order phase transition at the CP.

  - divergences in fluctuations of
    - p_T distribution
    - freezeout T
    - baryon number, chage, …



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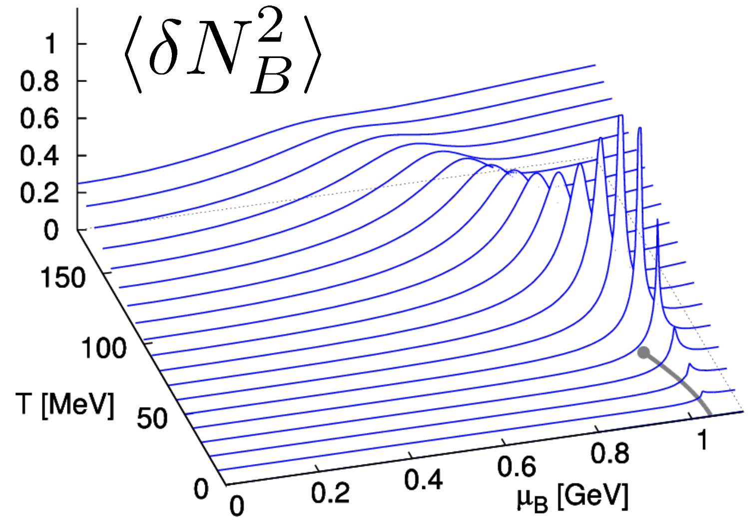


⇒
- divergences in fluctuations of
  - p_T distribution
  - freezeout T
  - baryon number, chage, …

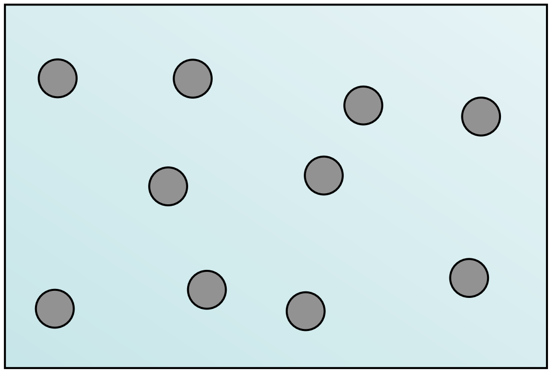
- Singular part in proton number fluctuations. Hatta, Stephanov, '02

$$\langle \delta N_p^2 \rangle \sim A\xi^2 + \langle \delta N_p^2 \rangle_{\mathrm{regular}}$$

- Higher order moments has stronger ξ dep near the CP. Stephanov, '09

$$\langle \delta N^2 \rangle \sim \xi^2 \quad \langle \delta N^3 \rangle = \xi^{4.5} \quad \langle \delta N^4 \rangle_c = \xi^7$$

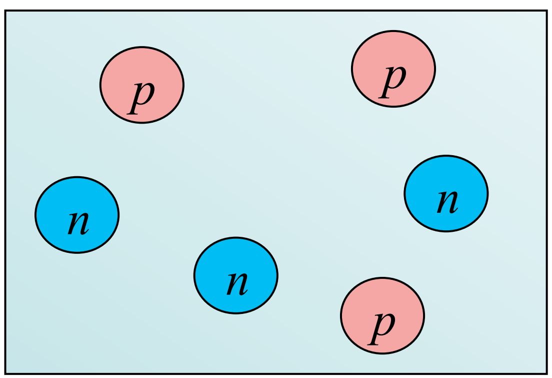
準粒子の素電荷とゆらぎ



Boltzmann gas $(T, \mu \ll M)$
(=Poisson distribution)

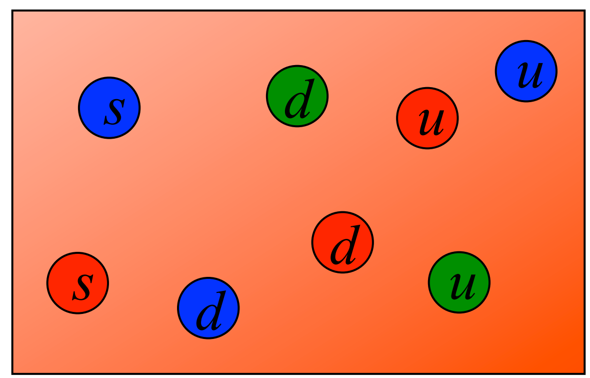
$$\langle N \rangle = \langle \delta N^2 \rangle = \langle \delta N^3 \rangle = \langle \delta N^4 \rangle_c = \cdots$$

Hadrons:



$$N_B = N$$

Quark-gluon:

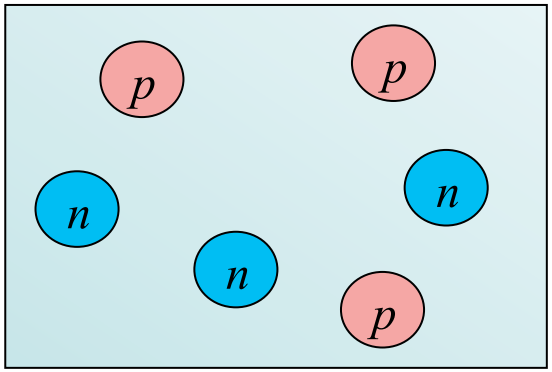


$$N_B = \frac{1}{3} N$$

準粒子の素電荷とゆらぎ

Asakawa, Heinz, Muller, '00
Jeon, Koch, '00
Ejiri, Karsch, Redlich, '06

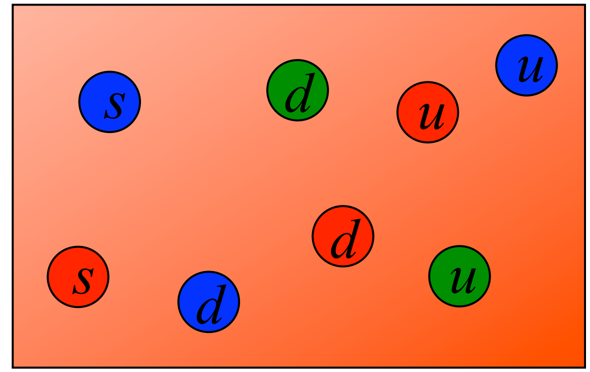
Hadrons: $N_B = N$



$$\frac{\langle \delta N_B^2 \rangle}{\langle N_B \rangle} = 1$$

$$\frac{\langle \delta N_B^n \rangle_c}{\langle N_B \rangle} = 1$$

Quark-gluon: $N_B = \frac{1}{3} N$



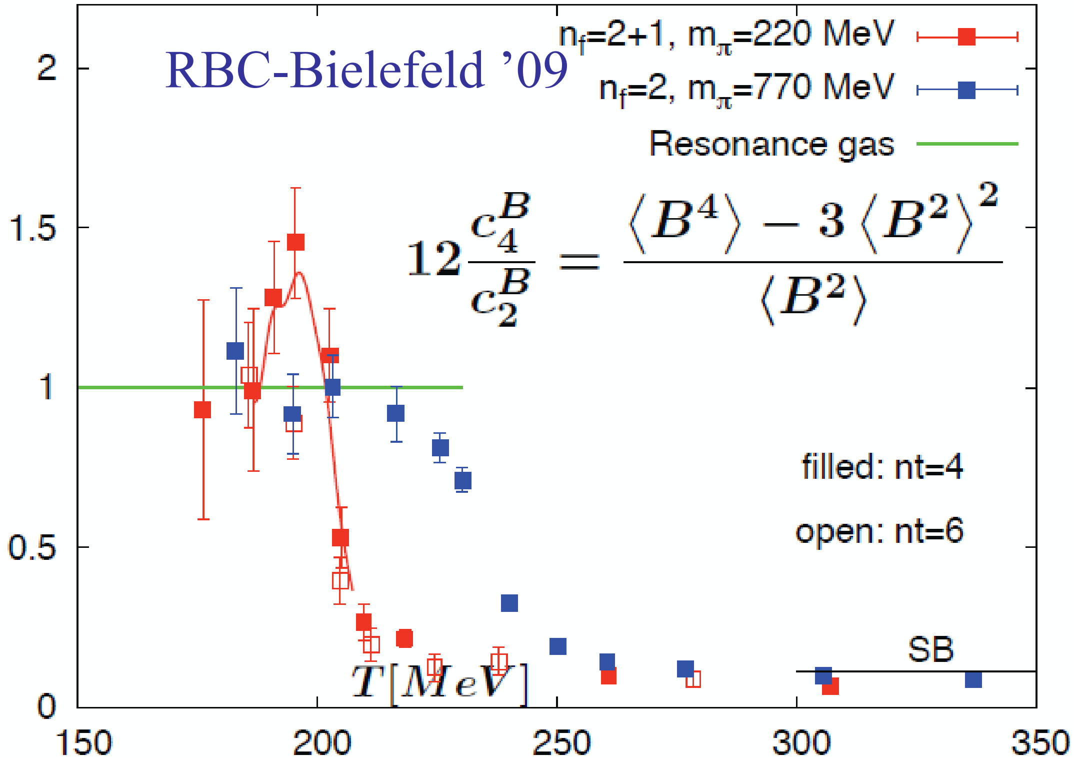
$$\frac{\langle \delta N_B^2 \rangle}{\langle N_B \rangle} = \frac{1}{3}$$

$$\frac{\langle \delta N_B^n \rangle_c}{\langle N_B \rangle} = \frac{1}{3^n}$$

Baryon Number 4th/2nd

Ejiri, Karsch, Redlich, '05

- Ratios between higher order moments (cumulants)

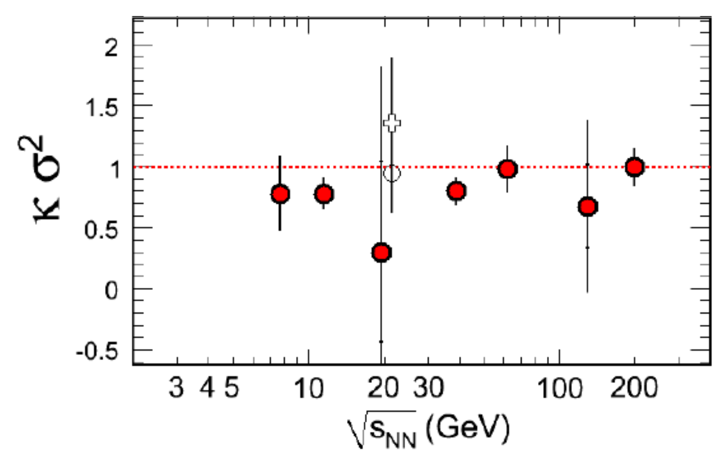


4th/2nd at μ=0 reflects the charge of quasi-particles.
For, $T \ll m, \ \ \kappa\sigma^2 \sim Q^2$

Hadrons:1 ⟺ Quarks:$1/3^2$

実験結果： c4/c2~1

STAR '10,'11



保存電荷の高次ゆらぎ 1

有限温度の期待値 $\langle \hat{O} \rangle = \sum_n e^{-\beta(E_n - \mu N_n)} \langle n | \hat{O} | n \rangle$

保存電荷の高次ゆらぎ 1

有限温度の期待値 $\langle \hat{O} \rangle = \sum_n e^{-\beta(E_n - \mu N_n)} \langle n | \hat{O} | n \rangle$

$$Z = \sum_n \langle n | e^{-\beta(H-\mu N)} | n \rangle$$

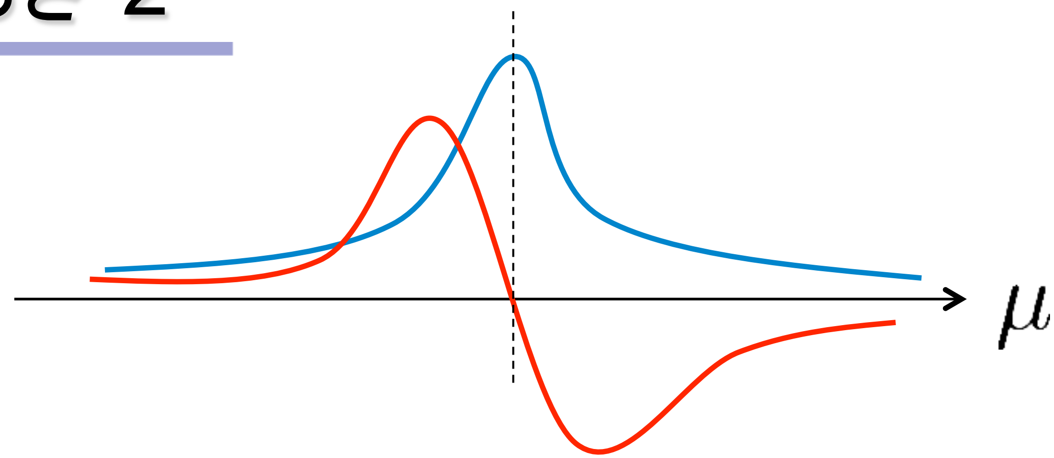
$$\Rightarrow \quad \frac{\partial Z}{\partial \mu} = \beta \sum_n \langle n | N e^{-\beta(H-\mu N)} | n \rangle = \langle N \rangle / T$$

$$\frac{\partial^2 \ln Z}{\partial \mu^2} = \langle \delta N^2 \rangle / T^2$$

$$\frac{\partial^3 \ln Z}{\partial \mu^3} = \langle \delta N^3 \rangle / T^3 = \frac{1}{T^2} \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$

保存電荷の高次ゆらぎ 2

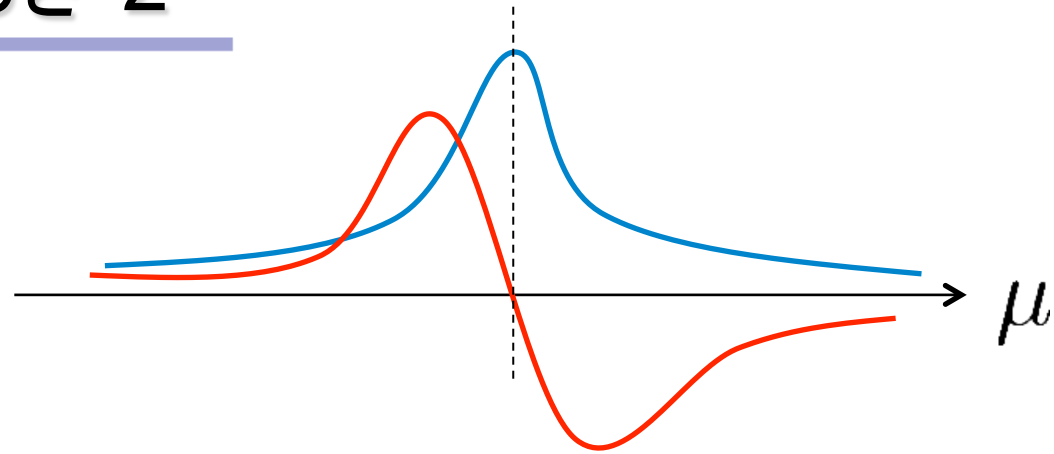
$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$



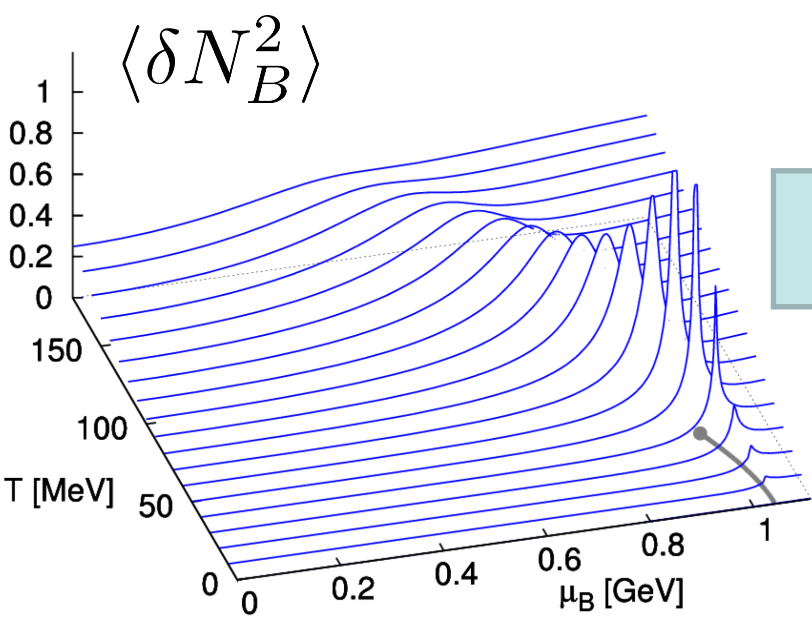
保存電荷の高次ゆらぎ 2

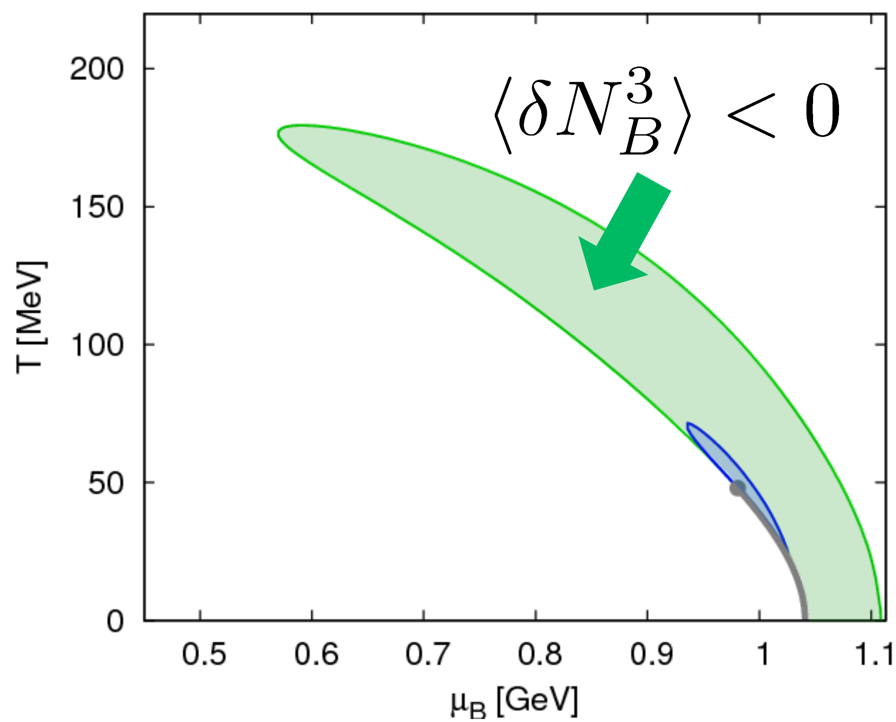
Asakawa, Ejiri, MK, 2009

$$\langle \delta N^3 \rangle = T \frac{\partial \langle \delta N^2 \rangle}{\partial \mu}$$

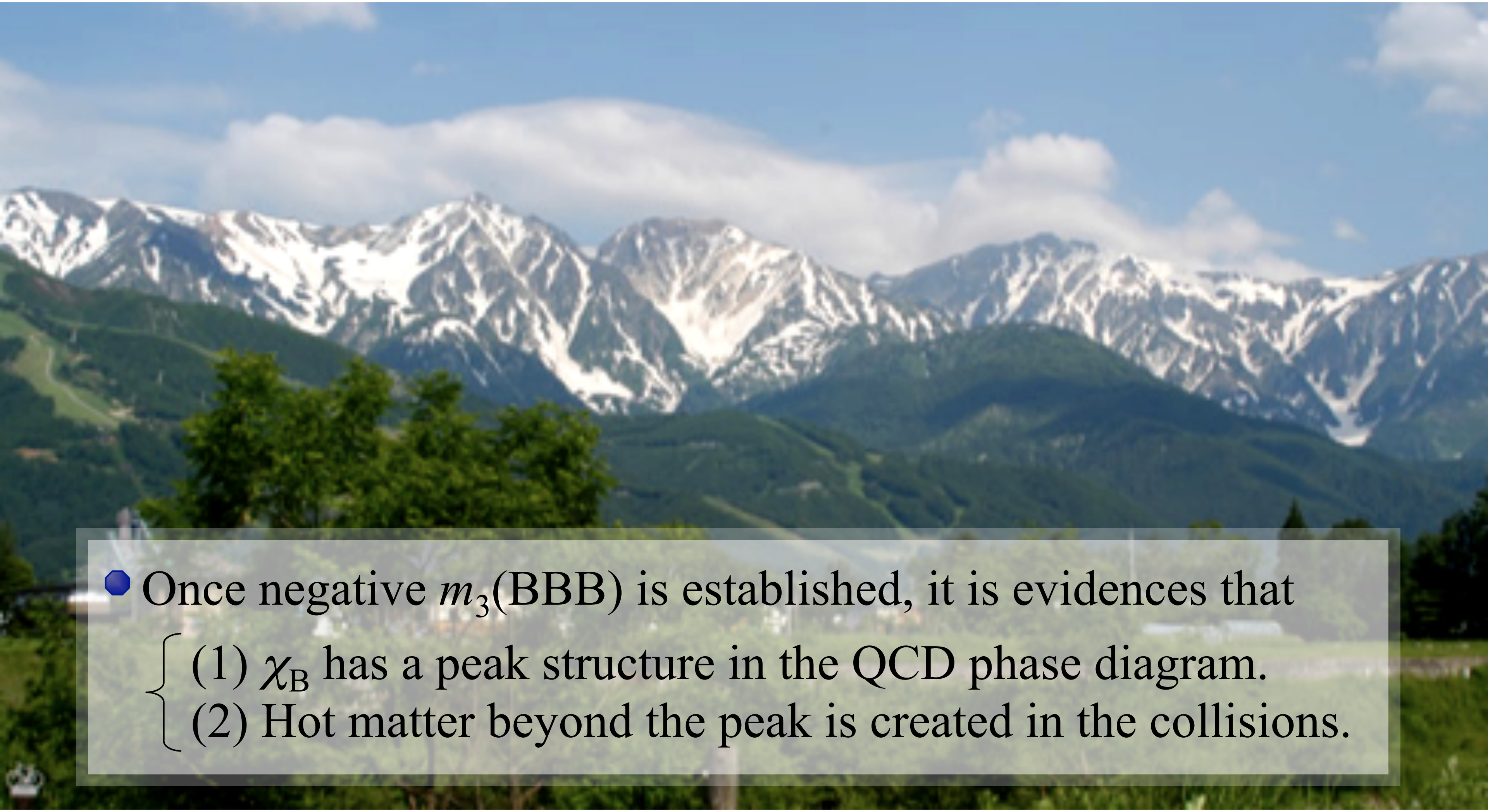


QCD相図上でのバリオン数ゆらぎ





Impact of Negative Third Moments

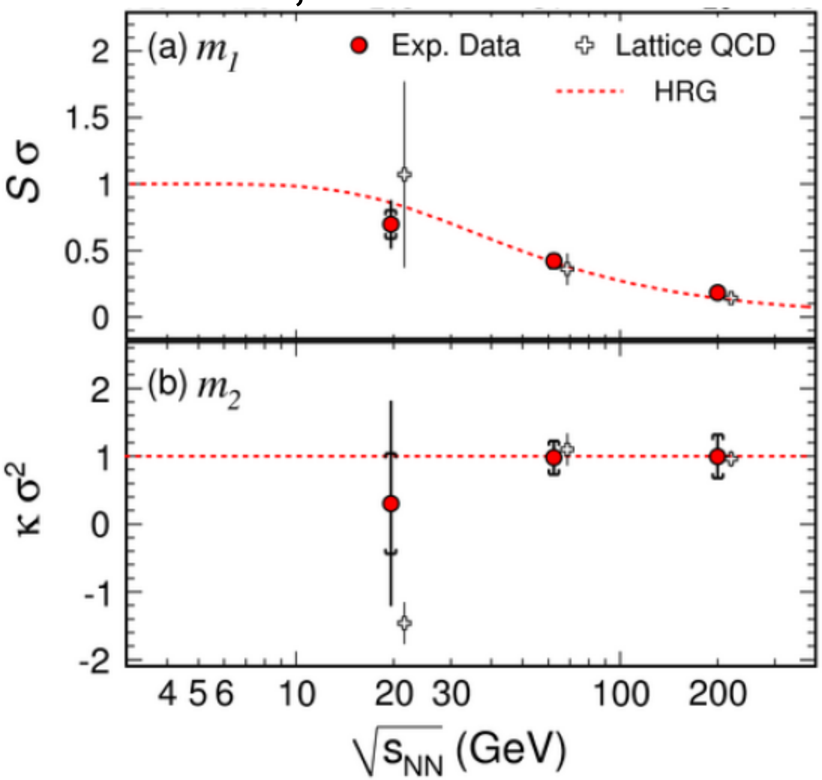


- Once negative m_3(BBB) is established, it is evidences that
  - (1) χ_B has a peak structure in the QCD phase diagram.
  - (2) Hot matter beyond the peak is created in the collisions.

-
  - •**No** dependence on any specific models.
  - •Just the sign! **No** normalization (such as by N_{ch}).

Proton # Fluctuations @ STAR

STAR, 2010



$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}, \quad \kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

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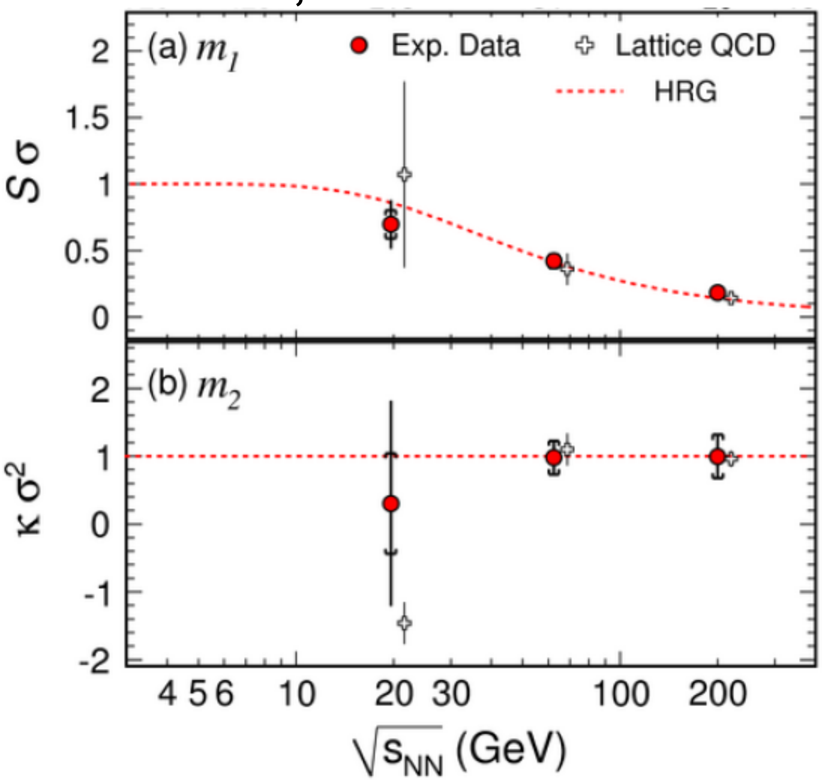
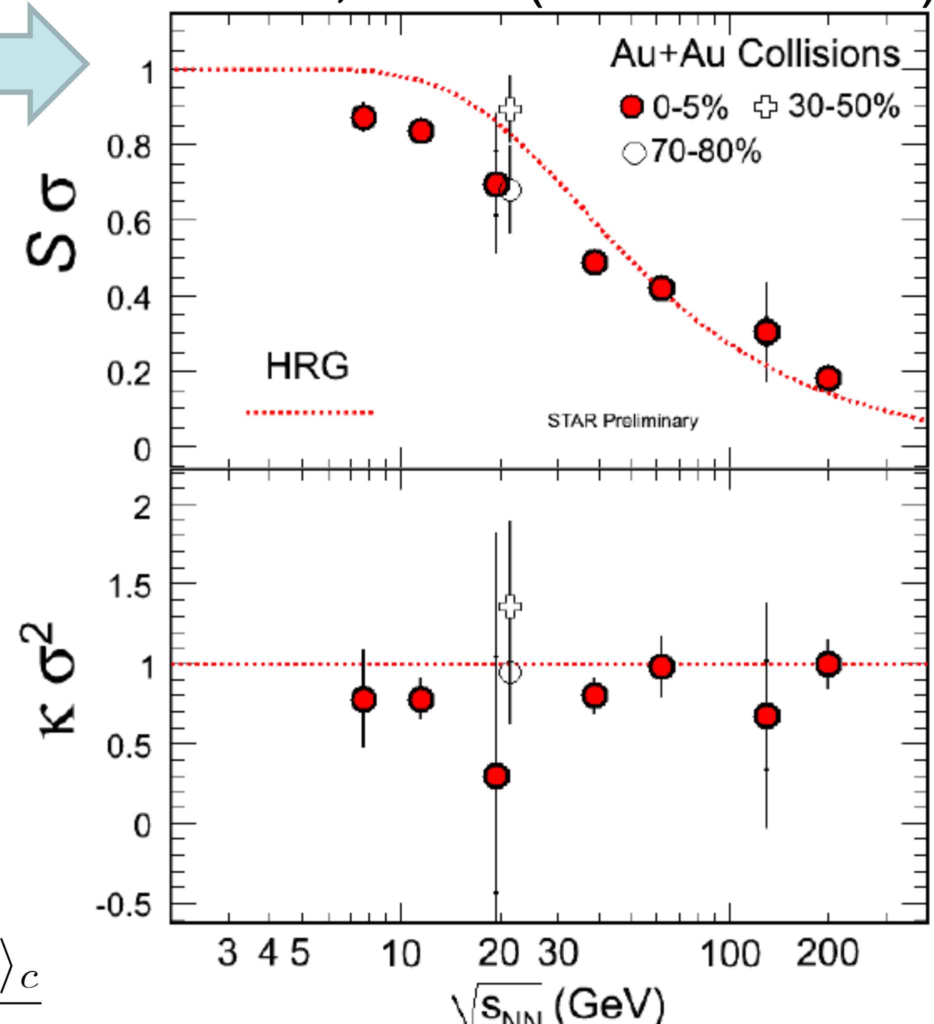


STAR, 2011 (Quark Matter)



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high μ ← → low μ

Proton # Fluctuations @ STAR

HRG:
Hadron Resonance Gas

=

Free gas composed of all hadrons & resonances

Poisson distribution for hadrons since $m_H \gg T, \mu$

$$S\sigma = \frac{\langle (\delta N_p^{(\mathrm{net})})^3 \rangle}{\langle (\delta N_p^{(\mathrm{net})})^2 \rangle}, \quad \kappa\sigma^2 = \frac{\langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c}{\langle (\delta N_p^{(\mathrm{net})})^2 \rangle}$$

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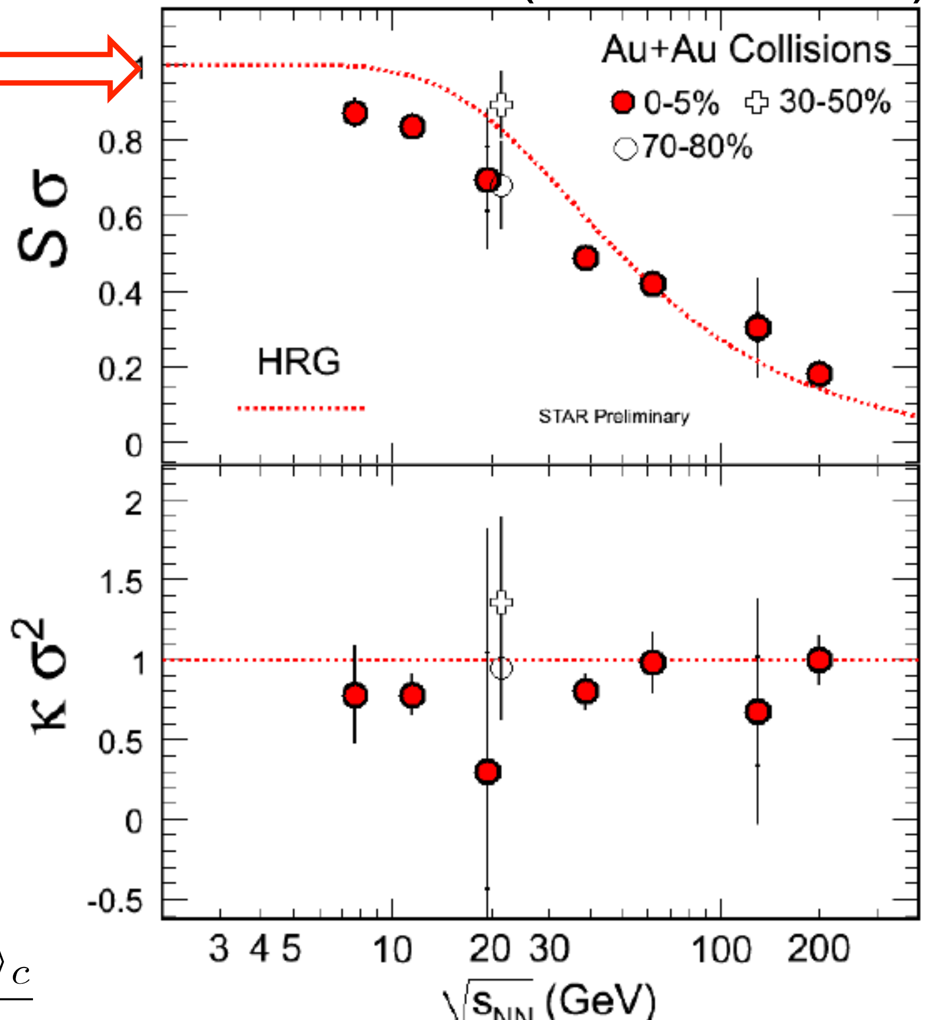
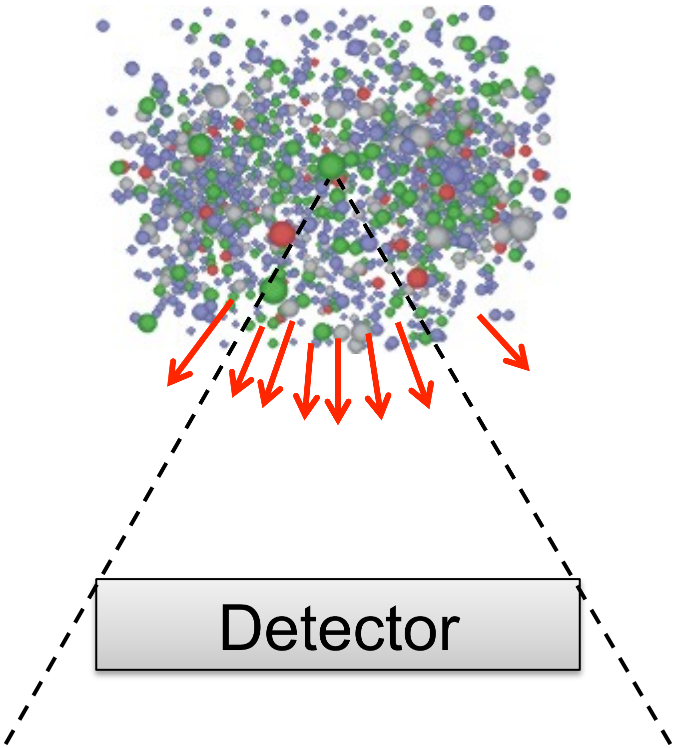


high μ ← → low μ

観測にかかるゆらぎは、いつ形成されたのか？

ゆらぎのダイナミクス（動的振る舞い）の議論が必要



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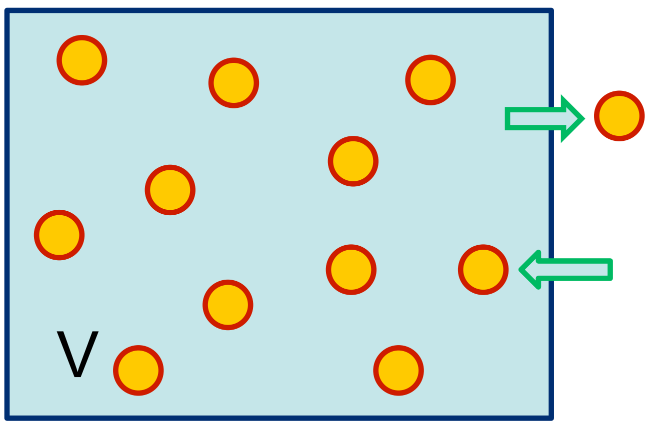
保存電荷の場合

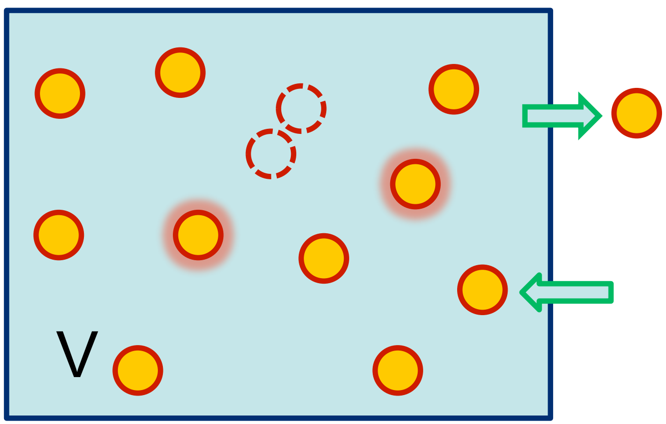


境界を通過する電荷のみが変化に寄与

$$\tau \to \infty$$

for $V \to \infty$

非保存電荷の場合



体積内の任意の場所で電荷が変化できる

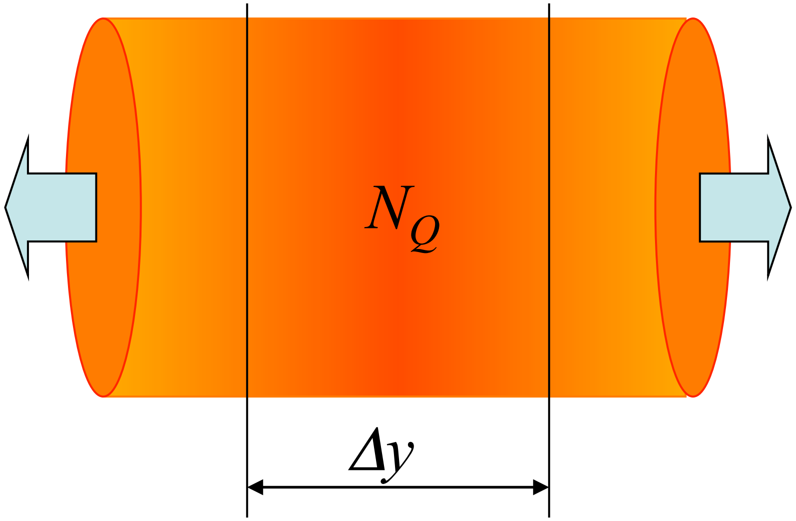
$$\tau \to \mathrm{const.}$$

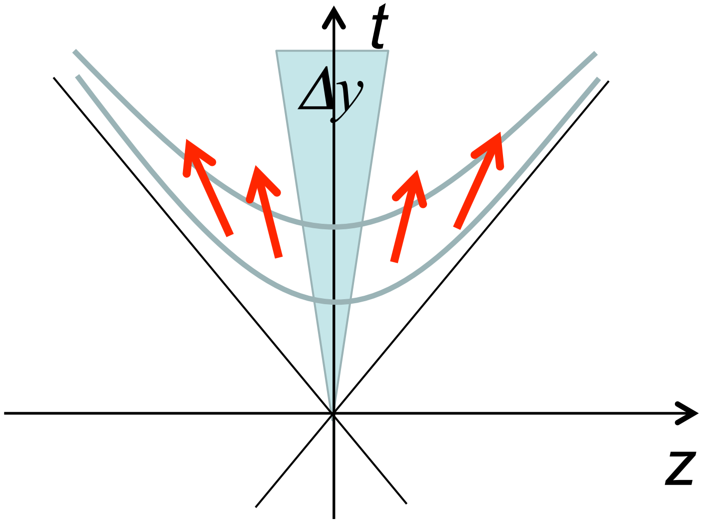
for $V \to \infty$

観測にかかるゆらぎは、いつ形成されたのか？

$\Delta\eta$内の保存電荷量は、初期段階のものが終状態まで生き残ることが期待できる。

Asakawa, Heinz, Muller, '00
Jeon, Koch, '00
Shuryak, Stephanov, '02





Note:

$$\text{STAR} \begin{cases} -0.5 < \eta < 0.5 \\ 0.4 < p < 0.8[GeV] \end{cases}$$

$$\langle \delta N_p^n \rangle \quad \Leftrightarrow ? \quad \langle \delta N_{\mathrm{B}}^n \rangle$$

- In equilibrated free nucleon gas,

$$\langle \delta N_{\mathrm{B}}^n \rangle_c = 2 \langle \delta N_p^n \rangle_c$$

- If the medium is not equilibrated,

$$\langle \delta N_{\mathrm{B}}^n \rangle_c \neq 2 \langle \delta N_p^n \rangle_c$$

Baryon Number Fluctuations are Better

- **than proton's** since it is not a conserved charge
  - simple theoretical treatment for conserved charges

$$\langle \delta N^n \rangle = \frac{\partial^n Z}{\partial \mu^n} \quad \Leftarrow \quad Z = tr[e^{-\beta(H-\mu N)}]$$

  - possible slow diffusion in hadronic stage

Asakawa, Heintz, Muller, 2000; Jeon, Koch, 2000

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Asakawa, Heintz, Muller, 2000; Jeon, Koch, 2000

- **than electric charge** fluctuations
  - additional nonsingular contribution in Q

Example: $\chi_Q = \frac{1}{4}\chi_B + \frac{1}{4}\chi_I$

(singular: $\frac{1}{4}\chi_B$; nonsingular: $\frac{1}{4}\chi_I$)

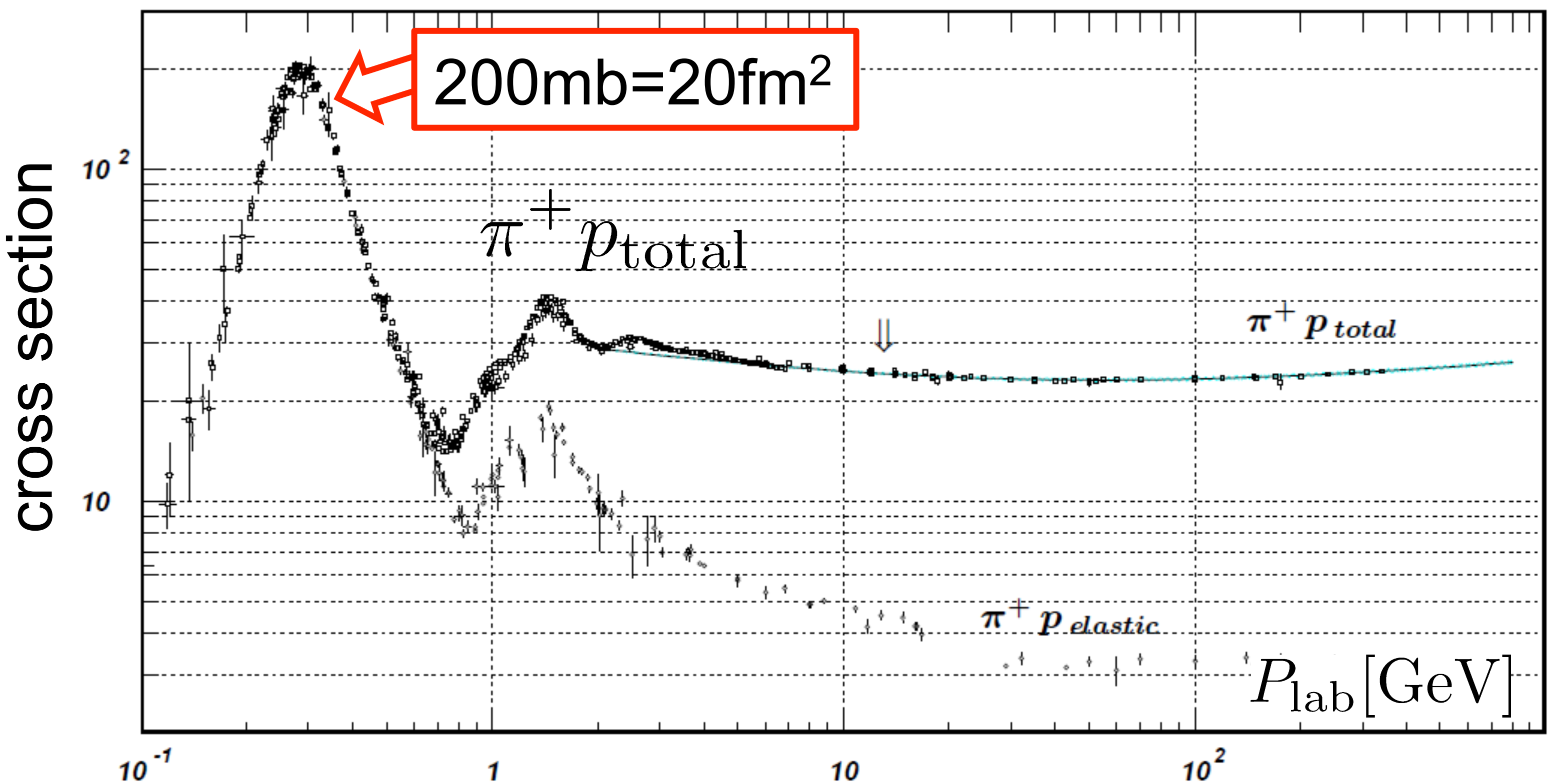
Variation of Proton # in Hadronic Phase

- Proton number varies even <u>after chemical freezeout</u> via charge exchange reactions mediated by Δ(1232):

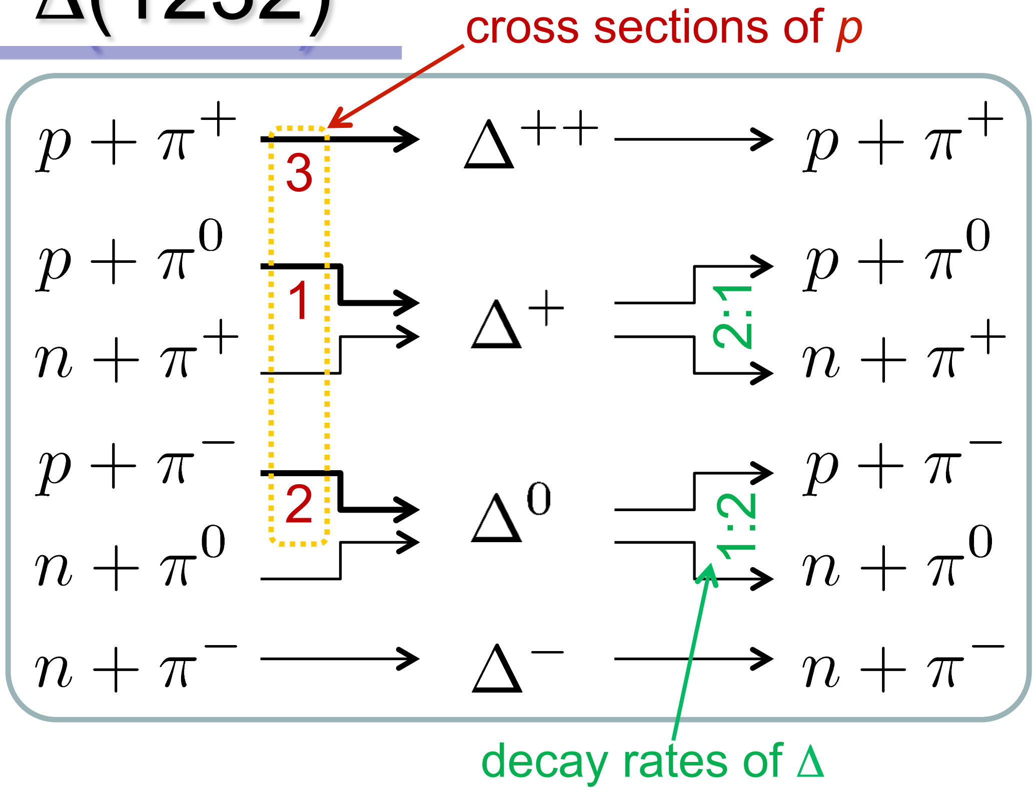
$$p,n \searrow \quad \Delta(1232) \quad \nearrow p,n$$
$$\pi \nearrow \qquad\qquad\qquad \searrow \pi$$

$I = 3/2$

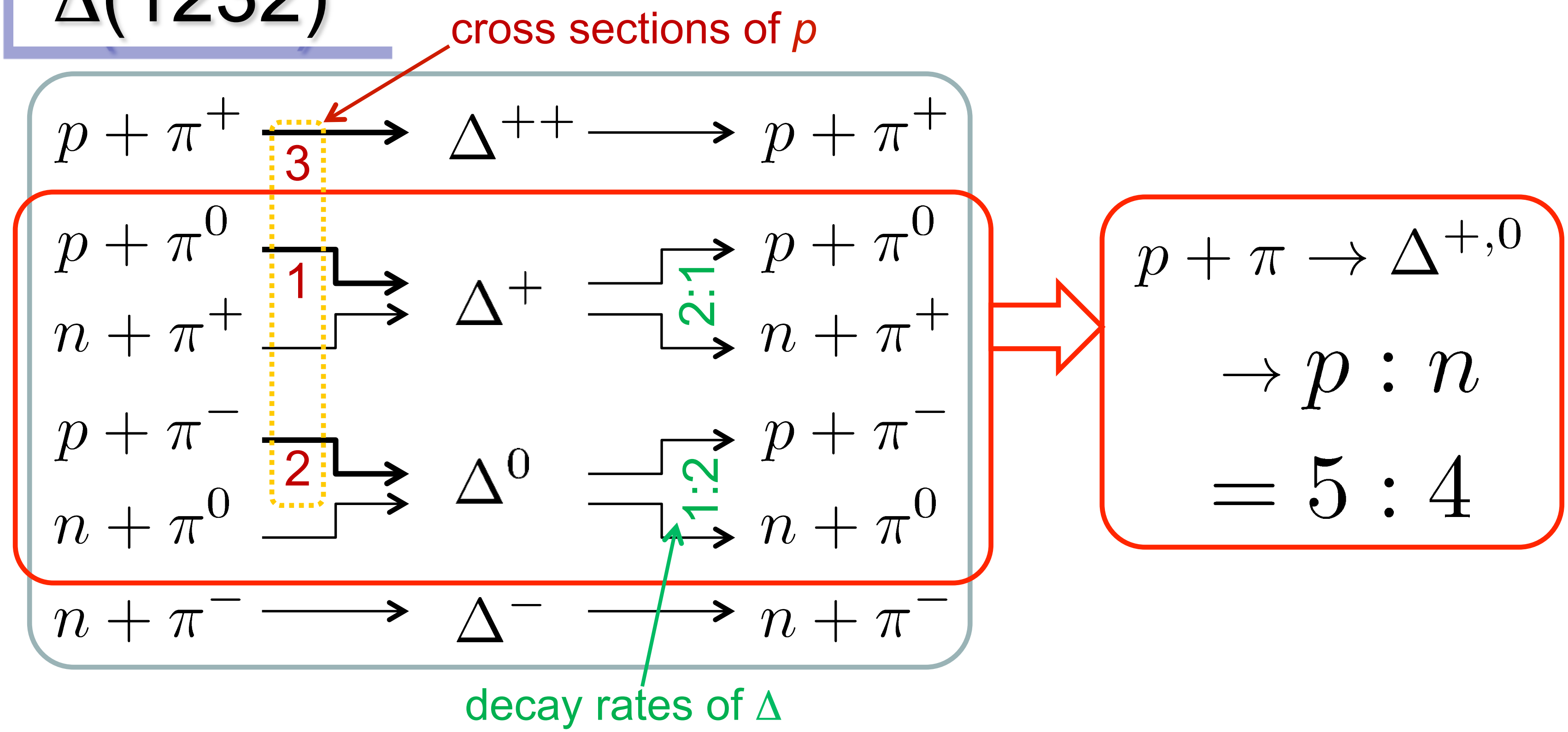
$\Gamma \simeq 1.8$ [fm]



Δ(1232)

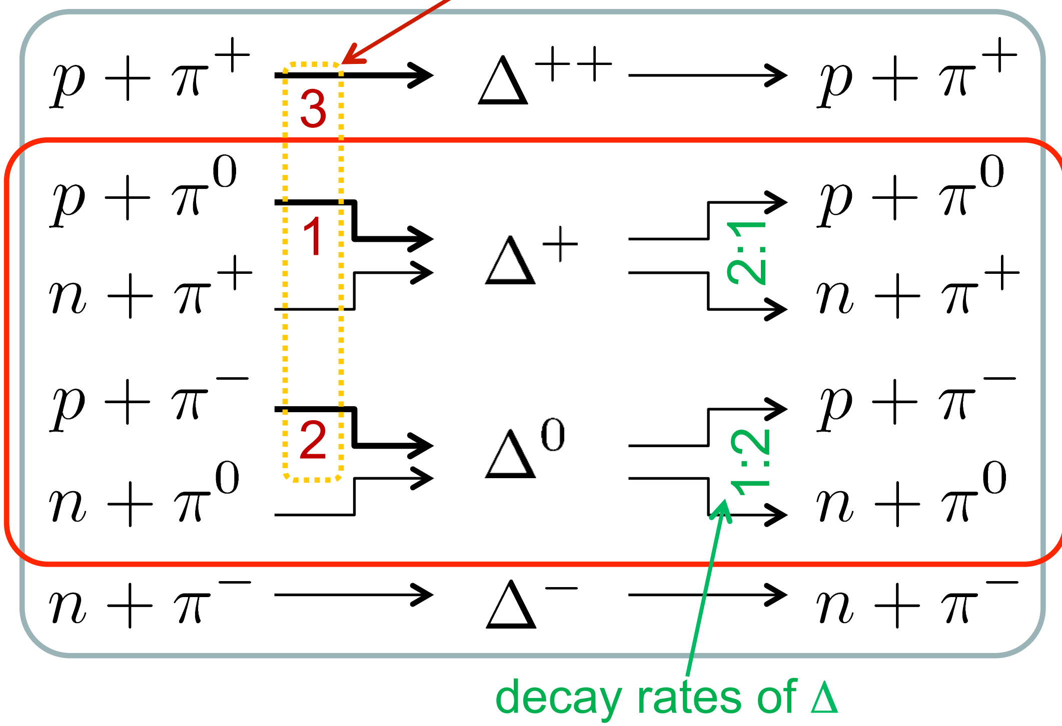

cross sections of p
p + π+
p + π0
n + π+
p + π−
n + π0
n + π−
3
1
2
Δ++
Δ+
Δ0
Δ−
p + π+
p + π0
n + π+
p + π−
n + π0
n + π−
2:1
1:2
decay rates of Δ

Δ(1232)



cross sections of p
p + π⁺ → Δ⁺⁺ → p + π⁺
3
p + π⁰
n + π⁺
1
Δ⁺
2:1
p + π⁰
n + π⁺
p + π⁻
n + π⁰
2
Δ⁰
1:2
p + π⁻
n + π⁰
n + π⁻ → Δ⁻ → n + π⁻
decay rates of Δ
p + π → Δ^{+,0}
→ p : n
= 5 : 4

Δ(1232)

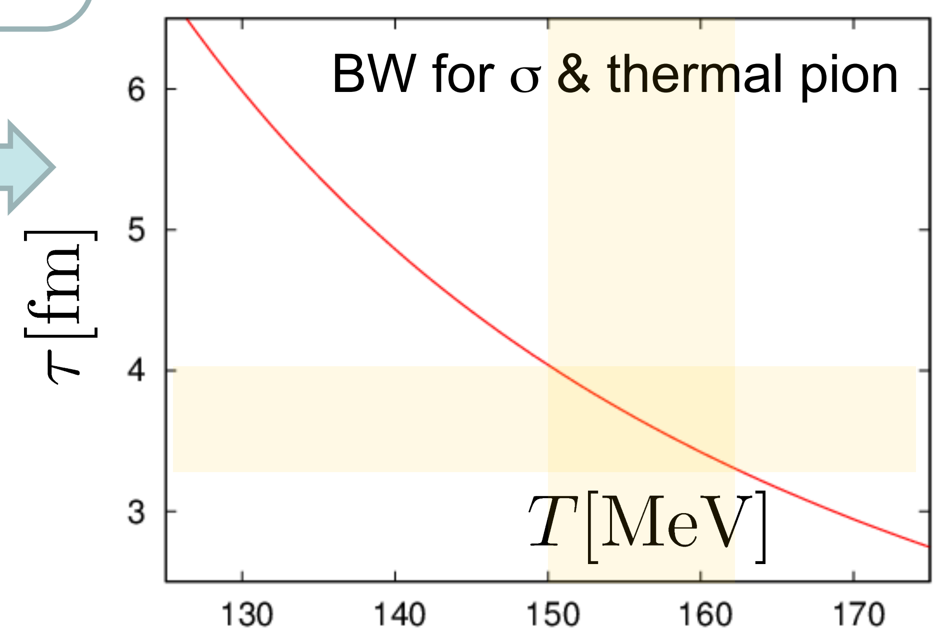


$$p+\pi \to \Delta^{+,0}$$
$$\to p : n$$
$$= 5 : 4$$

Lifetime to create Δ^+ or Δ^0

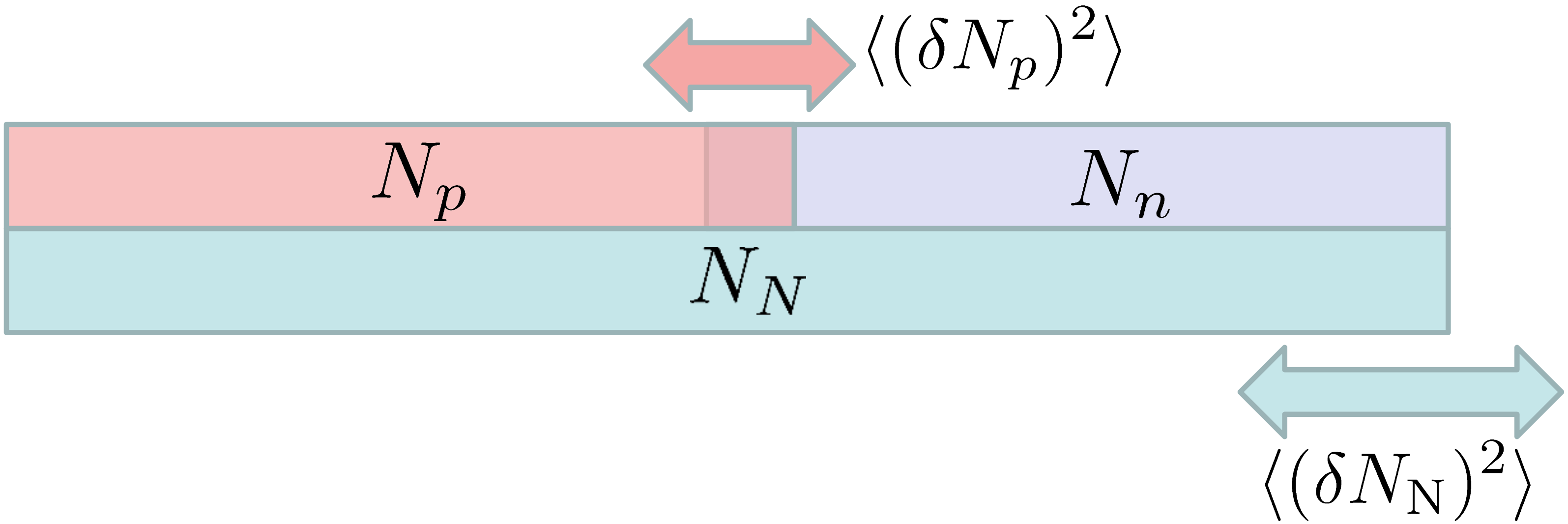
$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3}\sigma(E_{\rm cm})v_\pi n(E_\pi)$$

$$\tau \ll (\text{freezeout time}) \simeq 20[\text{fm}]$$



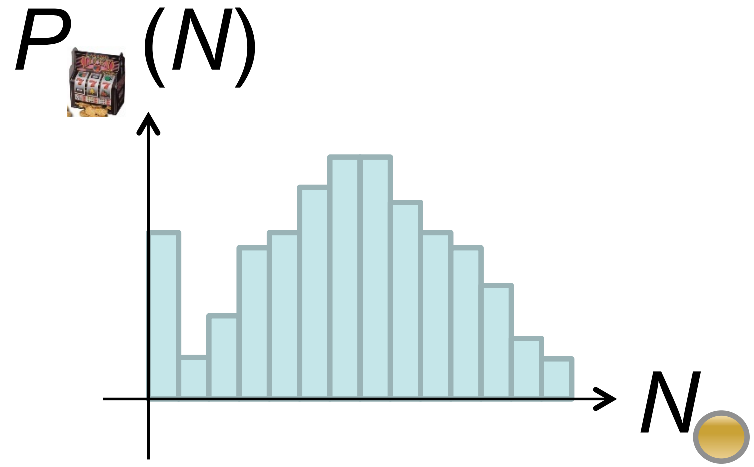
c.f.) Nonaka, Bass, 2007

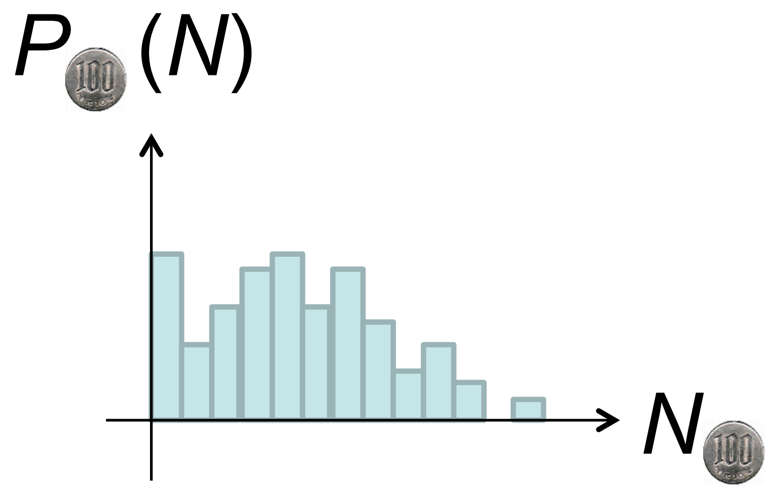
Baryon & Proton Number Fluctuations



- In general, fluctuations of N_N and N_p are different.
- Due to the isospin fluctuations, N_p fluctuations tend to be close to equilibrium ones than N_N fluctuations.

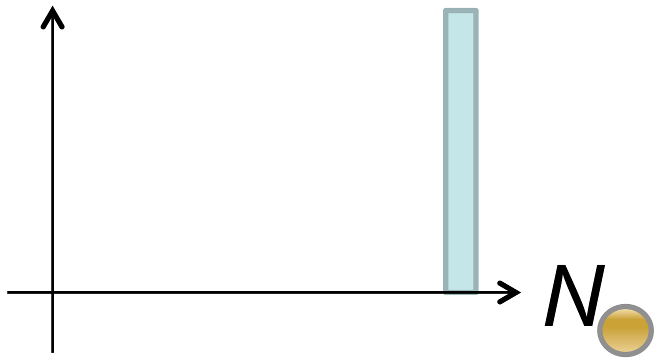
Slot Machine Analogy



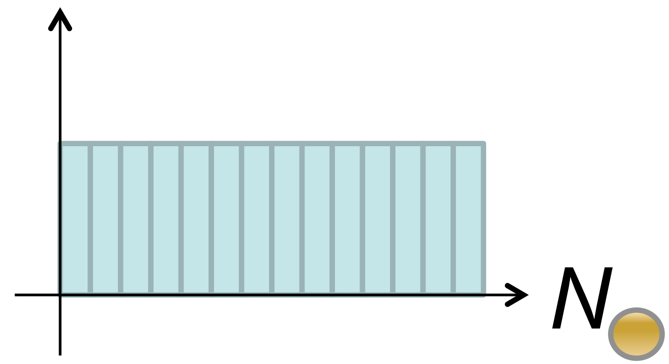


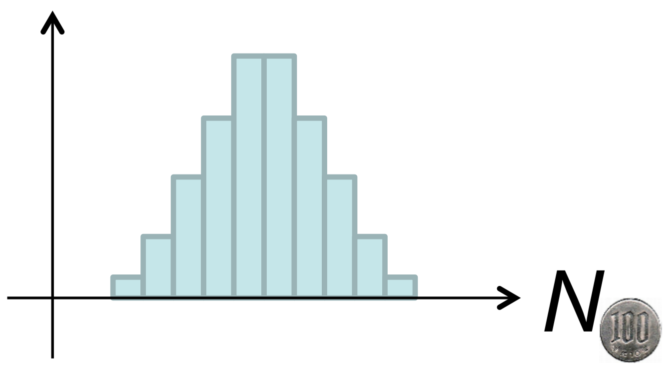
Examples

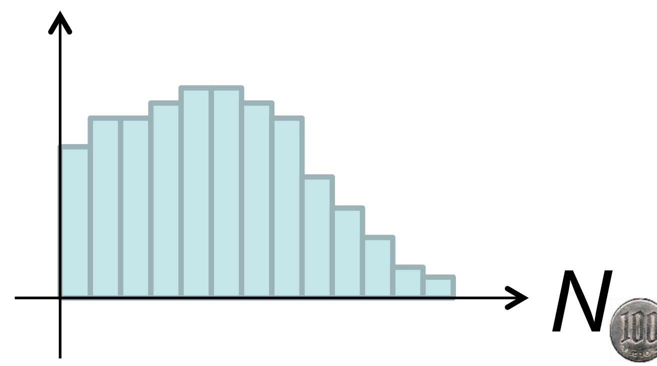
常に一定の枚数のコイン



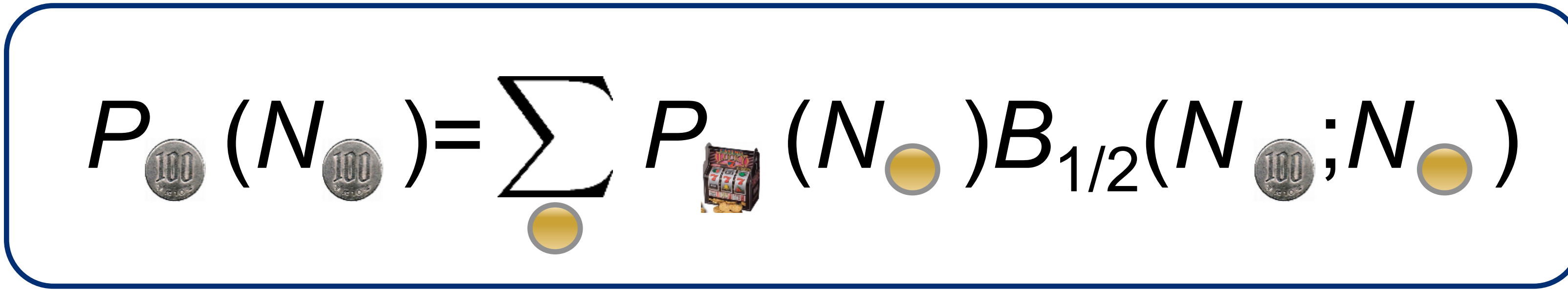
どの枚数も一定確率







Slot Machine Analogy



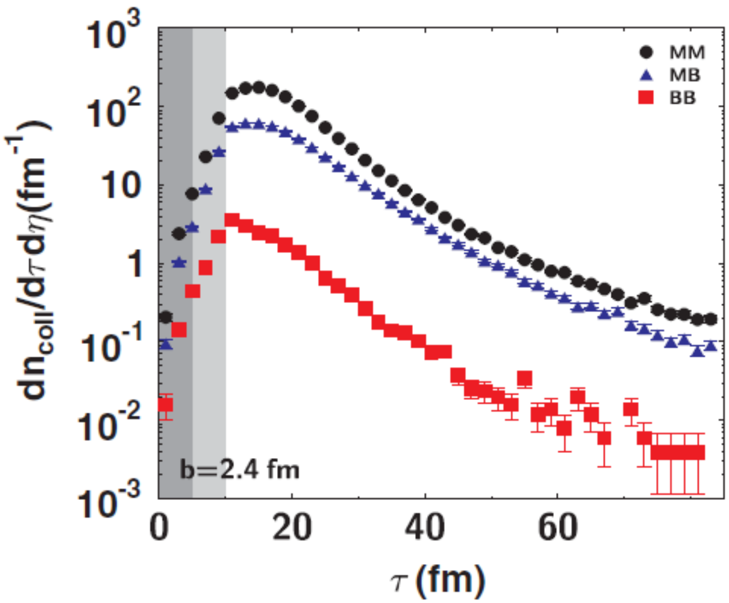
$$B_p(k;N) = p^k(1-p)^{N-k}\ {}_kC_N$$:二項分布関数

Isospin Distributions

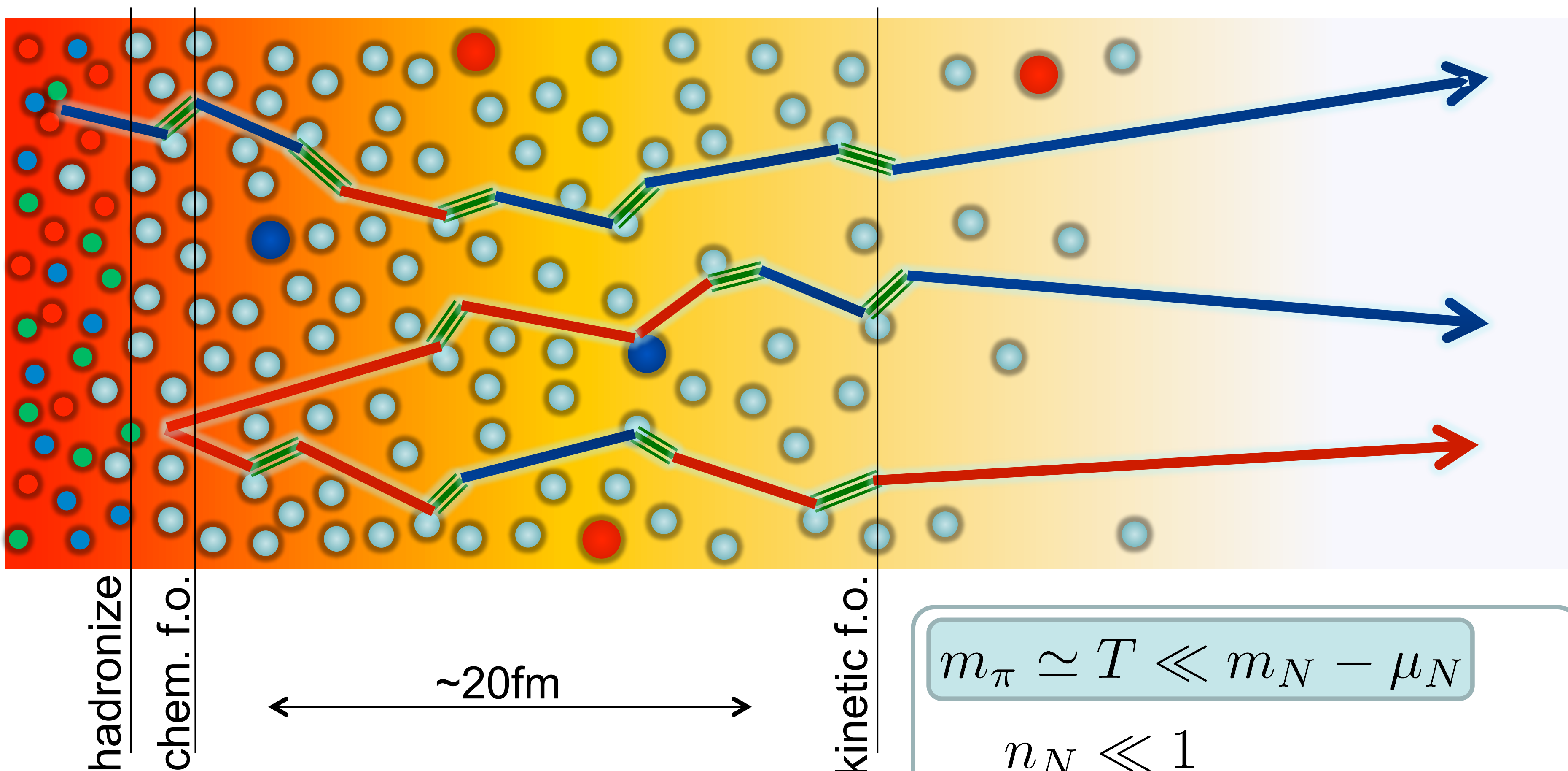
- Large pion density
  - Small nucleon density because $M_N/T \ll 1$
  - For top RHIC energy, $N_\pi \sim 20 N_N$
- Nucleons exclusively interact with pions
  - Rare NN collisions
  - Huge $\pi\pi$ reactions

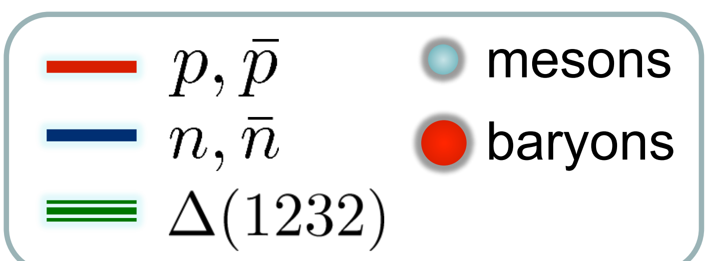
All formations and decays of Δ take place independently

Nonaka, Bass, '07



Nucleons in Hadronic Phase





$$m_\pi \simeq T \ll m_N - \mu_N$$

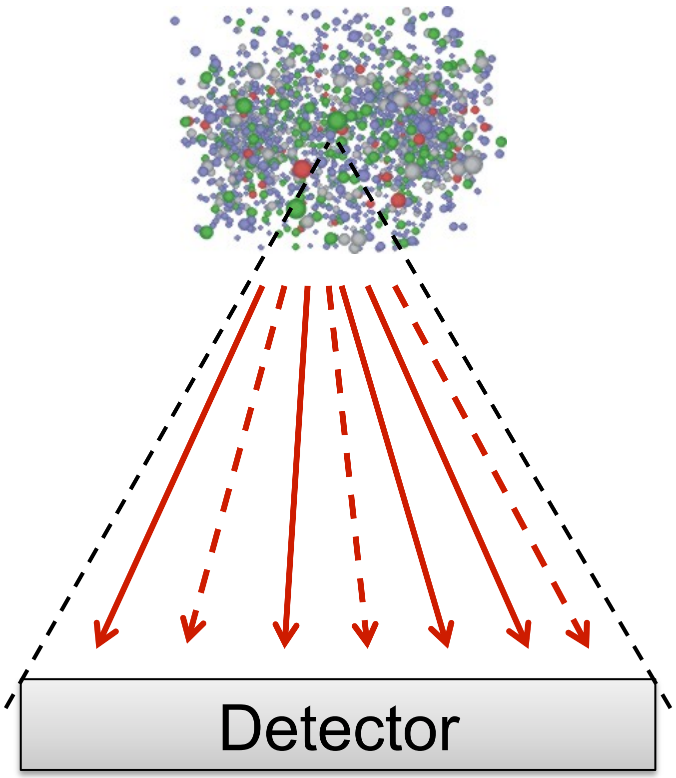
$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- many pions

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



- $\left\{ \begin{array}{l} \rightarrow N_N \text{ nucleons} \\ \dashrightarrow N_{\bar{N}} \text{ anti-nucleons} \end{array} \right.$

  ⇨ $F(N_N, N_{\bar{N}})$

- $N_N \left\{ \begin{array}{l} N_p \text{ protons} \\ N_n \text{ neutrons} \end{array} \right.$

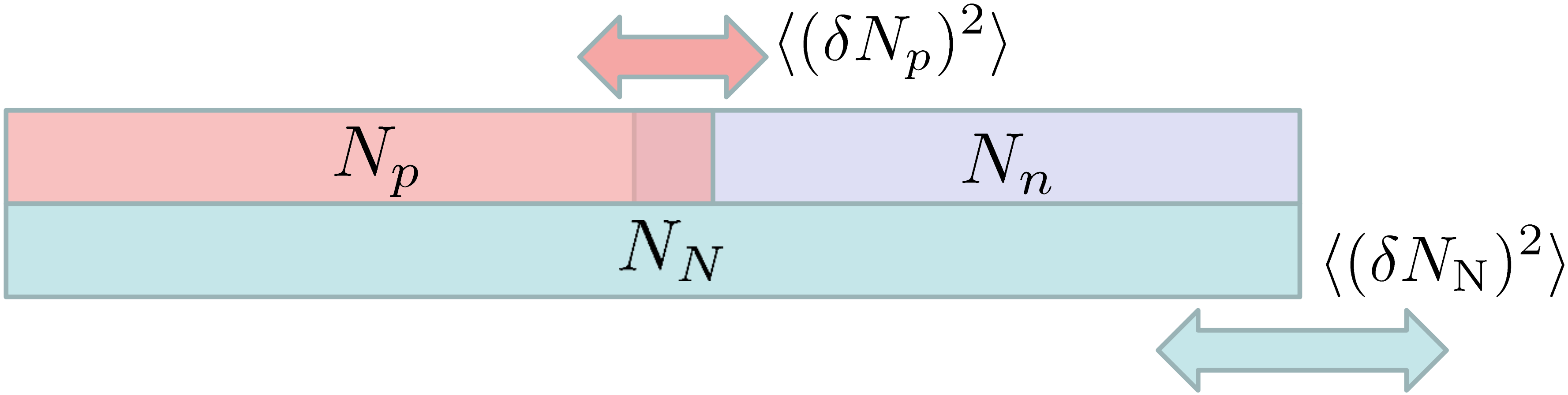
  ⇨ $B(N_p; N_N)$

binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

➢ for any phase space in the final state.

Baryon & Proton Number Fluctuations



$$
\begin{cases}
\langle(\delta N_p^{(\rm net)})^2\rangle = \frac{1}{4}\langle(\delta N_{\rm N}^{(\rm net)})^2\rangle + \frac{1}{4}\langle N_{\rm N}^{(\rm tot)}\rangle \\
\langle(\delta N_{\rm N}^{(\rm net)})^2\rangle = 4\langle(\delta N_p^{(\rm net)})^2\rangle - 2\langle N_p^{(\rm tot)}\rangle
\end{cases}
$$

- for isospin symmetric medium
- Similar formulas up to any order!

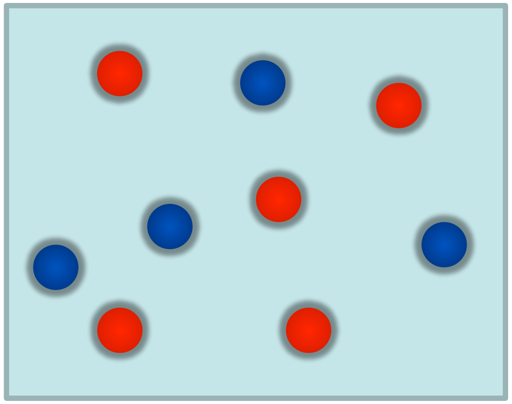
For free gas

$$\langle(\delta N_p^{(\rm net)})^2\rangle = \frac{1}{2}\langle(\delta N_{\rm N}^{(\rm net)})^2\rangle$$

Free Nucleon Gas

$T, \mu_{\mathrm{B}} \ll m_{\mathrm{N}}$ ⇨ Poisson distribution $P_\lambda(N)$

$$\mathcal{P}(N_p, N_n) = P_\lambda(N_p) P_\lambda(N_n)$$
$$= P_{2\lambda}(N_p + N_n) B_{1/2}(N_p; N_p + N_n)$$



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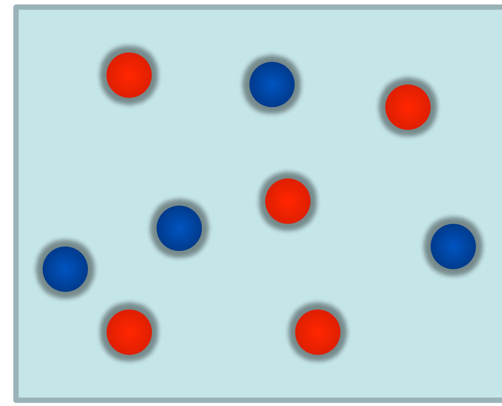


- The factrization is satisfied in free nucleon gas.

$$\mathcal{P}_{\rm free}(N_p, N_n, N_{\bar p}, N_{\bar n}) = \underbrace{P_{\bar N_{\rm N}}(N_{\rm N}) P_{\bar N_{\bar{\rm N}}}(N_{\bar{\rm N}})} B(N_p; N_{\rm N}) B(N_{\bar p}; N_{\bar{\rm N}})$$

$$F(N_{\rm N}, N_{\bar{\rm N}}) = P_{\bar N_{\rm N}}(N_{\rm N}) P_{\bar N_{\bar{\rm N}}}(N_{\bar{\rm N}})$$

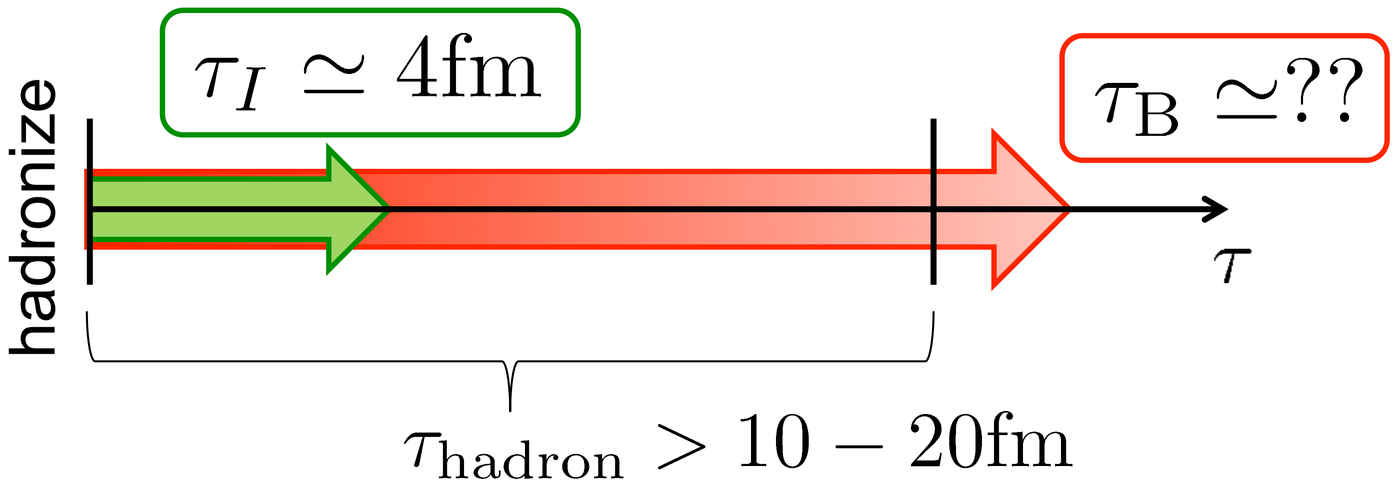
Time Scales

- Time scales of fireballs:

τ_I : time scale to realize isospin binomiality

$\tau_{\rm B}$: time scale of baryon number diffusion

$\tau_{\rm hadron}$: life-time of hadronic medium in HIC



Effect of Isospin Distribution

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

Effect of Isospin Distribution

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

$$
\left\{
\begin{aligned}
2\langle(\delta N_p^{(\mathrm{net})})^2\rangle &= \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^3\rangle &= \frac{1}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}} \\
2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c &= \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \frac{7}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_{c,\mathrm{free}}
\end{aligned}
\right.
$$

genuine info. noise

For free gas

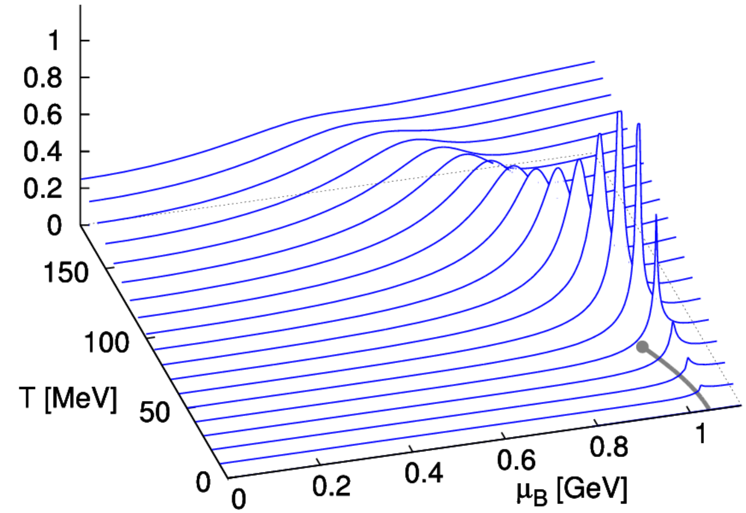
$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^n\rangle_c$$

Example : 3rd Moment

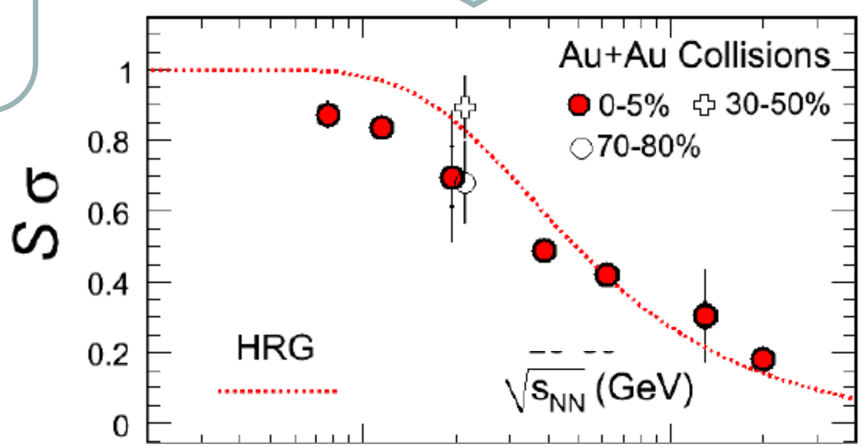
$$\langle(\delta N_{\rm B})^3\rangle = \frac{\partial\langle(\delta N_{\rm B})^2\rangle}{\partial\mu} < 0$$

beyond the QCD phase boundary
near the CP

Asakawa, Ejiri, MK, 2009



Fireballs forget negative moment over the QCD mountains?
No. Not necessarily.



Note:

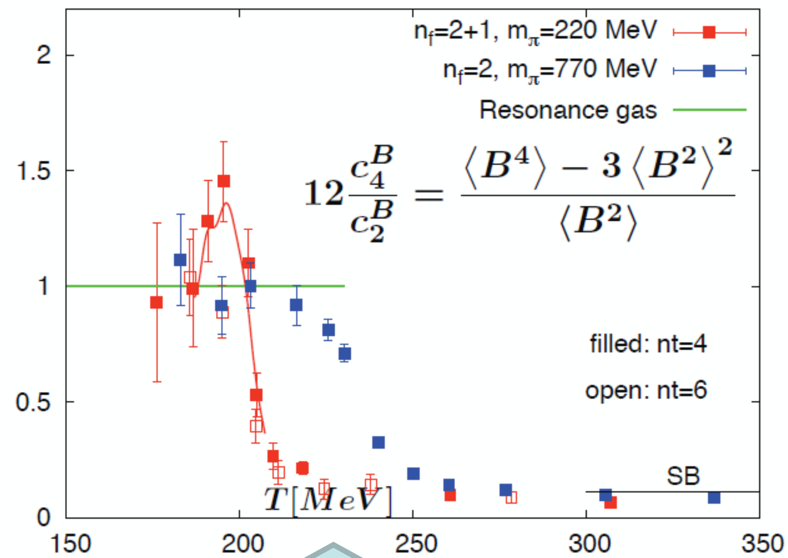
$$\begin{aligned}\langle(\delta N_{\rm B}^{(\rm net)})^3\rangle =& 8\langle(\delta N_p^{(\rm net)})^3\rangle \\ &- 12\langle\delta N_p^{(\rm net)}\delta N_p^{(\rm tot)}\rangle \\ &+ 6\langle N_p^{(\rm net)}\rangle,\end{aligned}$$

Example1 : 4th/2nd

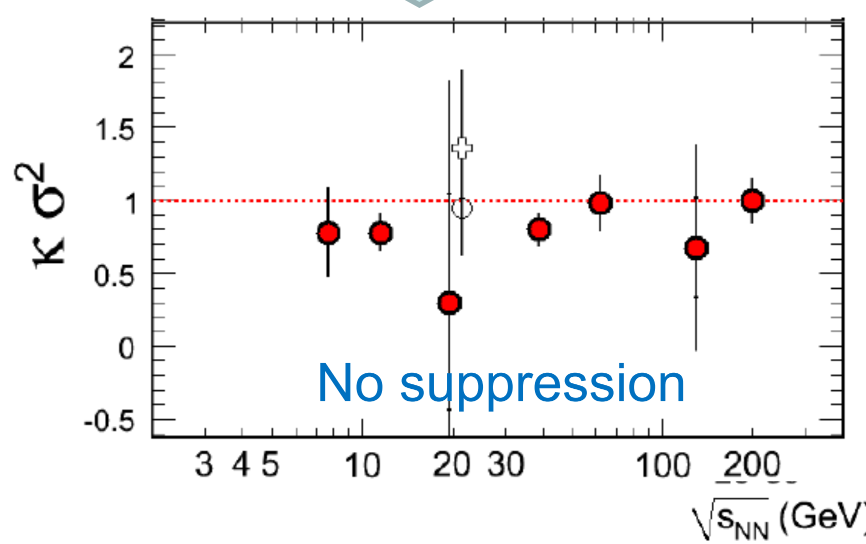
$$\frac{\langle(\delta N_{\rm B})^4\rangle_c}{\langle(\delta N_{\rm B})^2\rangle} = \begin{cases} \sim 1 \text{ (hadron)} \\ \sim 1/9 \text{ (quark)} \\ < 0 \text{ (near CP)} \end{cases}$$

Ejiri, Karsch, Redlich, 2006
Stephanov, 2011

RBC-Bielefeld '09



Fireballs forget primordial flucs?
No. Not necessarily.



Strange Baryons

Decay Rates:

Λ $\quad m_\Lambda \simeq 1116[\mathrm{MeV}]$

$\Rightarrow$ $p : n \simeq 1.6 : 1$

Σ $\quad m_\Sigma \simeq 1190[\mathrm{MeV}]$

$\Rightarrow$ $p : n \simeq 1 : 1.8$

Decay modes:

$\Lambda \to p + \pi^-$ 64%

$\Lambda \to n + \pi^0$ 36%

$\Sigma^+ \to p + \pi^0$ 52%

$\Sigma^+ \to n + \pi^+$ 48%

$\Sigma^0 \to \Lambda \to p + \pi^-$ 64%

$\Sigma^0 \to \Lambda \to n + \pi^0$ 36%

$\Sigma^- \to n + \pi^-$

Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \to N_B$

Summary

- **Baryon and proton number fluctuations are different** in nonequilibrium medium. To see non-thermal contribution, baryon number is better.

- Formulas to reveal baryon # cumulants in experiments.

- Experimental analysis of baryon # fluctuations may verify
  - signals of QCD phase transition
  - speed of baryon number diffusion in the hadronic stage.

Future Work

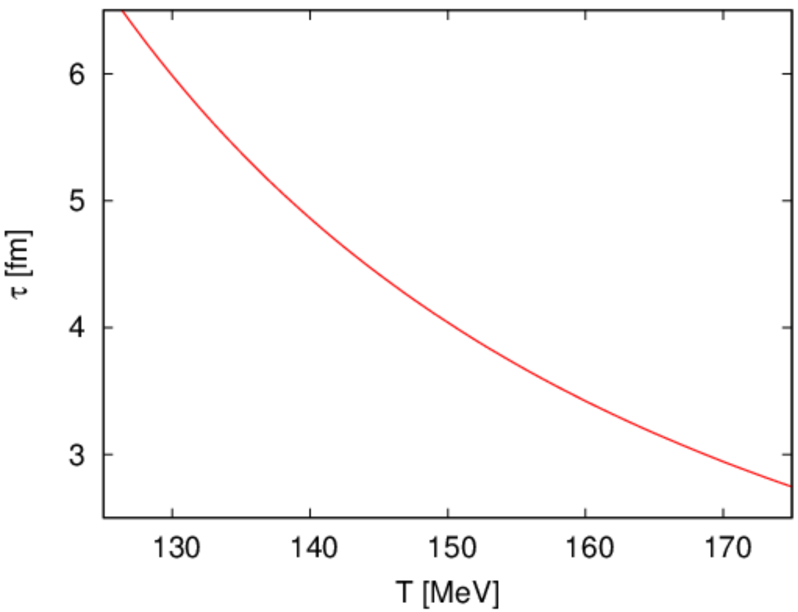
- Refinement of the formulas to incorporate
  nonzero isospin density / low beam-energy region
- Distribution function itself
- Discussion on time evolution of fluctuations @STAR

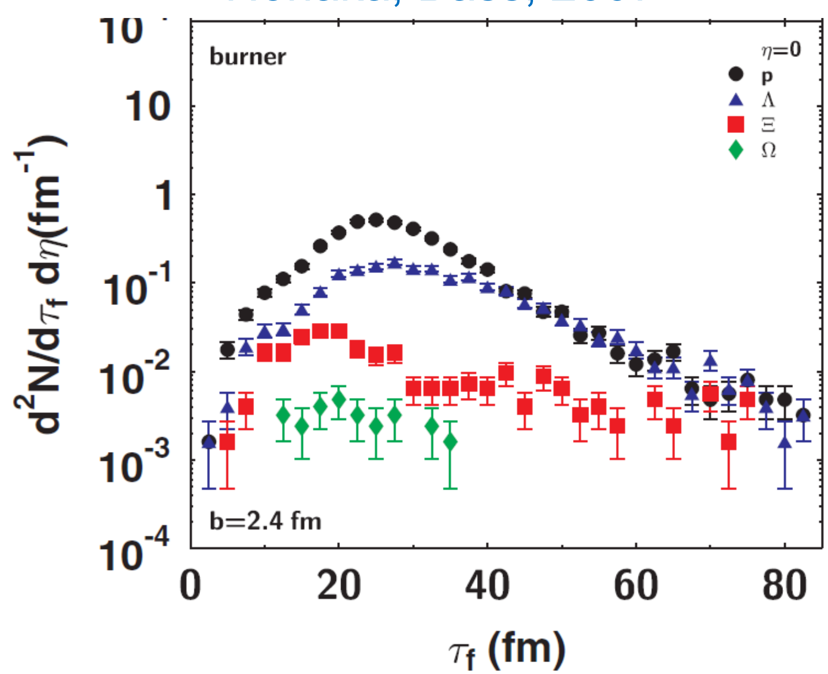
Nucleon Time Scales in Fireballs

Freeze-out time

Mean time to create $\Delta^{+,0}$

Nonaka, Bass, 2007





$$\tau_\Delta = 3 \sim 4[\mathrm{fm}]$$

$$\tau_{\mathrm{f.o.}} > 20[\mathrm{fm}]$$

3rd & 4th Order Fluctuations

$N_{\mathrm{B}} \to N_p$

$$\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{8}\langle\delta N_{\mathrm{B}}^{(\mathrm{net})}\delta N_{\mathrm{B}}^{(\mathrm{tot})}\rangle,$$

$$\begin{aligned}\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c =& \frac{1}{16}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \frac{3}{8}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\delta N_{\mathrm{B}}^{(\mathrm{tot})}\rangle \\ &+ \frac{3}{16}\langle(\delta N_{\mathrm{B}}^{(\mathrm{tot})})^2\rangle - \frac{1}{8}\langle N_{\mathrm{B}}^{(\mathrm{tot})}\rangle,\end{aligned}$$

$N_p \to N_{\mathrm{B}}$

$$\begin{aligned}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle =& 8\langle(\delta N_p^{(\mathrm{net})})^3\rangle - 12\langle\delta N_p^{(\mathrm{net})}\delta N_p^{(\mathrm{tot})}\rangle \\ &+ 6\langle N_p^{(\mathrm{net})}\rangle,\end{aligned}$$

$$\begin{aligned}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c =& 16\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c - 48\langle(\delta N_p^{(\mathrm{net})})^2\delta N_p^{(\mathrm{tot})}\rangle \\ &+ 48\langle(\delta N_p^{(\mathrm{net})})^2\rangle + 12\langle(\delta N_p^{(\mathrm{tot})})^2\rangle - 26\langle N_p^{(\mathrm{tot})}\rangle,\end{aligned}$$